MATH 113, HOMEWORK #12

Due Thursday, November 19

Remember, consult the Homework Guidelines for general instructions. Keep in mind that for us, rings are always required to have a multiplicative identity.

GRADED HOMEWORK:

1. Mimic Section 29, Exercise #25, but with the polynomial $x^3 + 2x + 1$ in $\mathbb{Z}_3[x]$.
2. Build a field F with 8 elements by taking an appropriate factor ring of $R = \mathbb{Z}_2[x]$. Use results from recent sections to prove your factor ring is a field AND also explicitly find the inverse of each nonzero element.

   (Think about, but don't turn in: for your particular F, what group is $\langle F, +\rangle$ isomorphic to? What group is $\langle F^*, \cdot\rangle$ isomorphic to?)
3. Suppose F is a finite field of characteristic p, where p is prime.
   (a) Prove that the order of F is p^k, for some integer k.
   (b) Let P denote the prime subfield of F. (Note this is the smallest nontrivial subfield of F, and it is isomorphic to $\mathbb{Z}_p$.) Prove that every element of F is algebraic over P, i.e. every $\alpha \in F$ is the root of a polynomial in $P[x] \cong \mathbb{Z}_p[x]$.

UNGRADED HOMEWORK:

Note: Starred problems from this list are classic results you will almost certainly need to use again.

Section	Problems
29	# 24, 25, 26, 30, 31, 34, 36
31	# 22, 23*, 24, 25, 29*