MATH 113, HOMEWORK #9

Due Thursday, October 29

Remember, consult the Homework Guidelines for general instructions. Keep in mind that for us, rings are always required to have a multiplicative identity.

GRADED HOMEWORK:

1. Let $G = GL(7, \mathbb{R})$. We have already showed that the subset $N = \{M \in G : \det M = 1\}$ is a subgroup of G. Verify that $N \trianglelefteq G$ and describe the cosets of N. Determine the isomorphism type of G/N and prove your answer.

2. Let $R = \mathbb{R}[[x]]$ denote the ring of formal power series in the variable x with real coefficients. I.e. $R = \{a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots | a_i \in \mathbb{R}\}$. We add and multiply formal power series just as we do polynomials. The main difference is a formal power series may have infinitely many nonzero terms (though the polynomials do belong to R). Note that *formal* indicates we are not thinking of these as functions we will plug points into; therefore convergence (or lack thereof) is a nonissue for us.

   You may take it as a given that R is a commutative ring. (It is not especially difficult to show, but I don't need you to write it up carefully.)

   (a) Prove that R is an integral domain.

   (b) Find a nonzero element of R which is neither a unit nor a zero divisor. Prove your answer meets this condition.

   (c) Find the inverse of the linear polynomial $ax - 1$ in R where a is a nonzero constant.

3. Let $R = M_2(\mathbb{Z}_6)$, the ring of 2×2 matrices with entries from $\mathbb{Z}_6$.

   (a) How many elements are in R? What can you say about the orders of subrings of R? Why?

   (b) As completely as you can, classify the nonzero elements of R as units, zero divisors, or neither.

   (c) If R were an integral domain, we'd be guaranteed at most two solutions to the equation

   $$X^2 - 4X + 3I = 0.$$

   Can you find more than two?

UNGRADED HOMEWORK:

Starred problems from this list are classic results you will almost certainly need to use again.

Section	Problems
19	17, 24*, 27, 29*
20	6, 11-20, 27-29