MATH 113, HOMEWORK #8

Due Thursday, October 22

Remember, consult the Homework Guidelines for general instructions.

GRADED HOMEWORK:

1. Determine the isomorphism type of the factor group $\mathbb{Z}_8 \times \mathbb{Z}_6 \times \mathbb{Z}_4/\langle(2,2,2)\rangle$ and give two proofs – one using elementary analysis of the orders of elements, and one using the First Isomorphism Theorem.

2. Let $R = M_n(\mathbb{R})$ denote the ring of $n \times n$ matrices with real entries. Determine (with proof) whether each of the following is a subring of R. (Recall that for us, every ring and thus every subring must have a multiplicative identity.)

   (a) $T = \{A \in R : trace(A) \in \mathbb{Q}\}$

   (b) $D = \{A \in R : det(A) \in \mathbb{Q}\}$.

   (c) $L = \{A \in R : A \text{ is lower triangular}\}$

3. Let $R = \mathbb{Z}[x]$, the ring of polynomials with coefficients in $\mathbb{Z}$. The following parts are not related to each other.

   (a) Let $S = \{p(x) \in R : \text{ every term of } p(x) \text{ has even degree}\}$
   $= \{a_0 + a_2x^2 + a_4x^4 + \cdots a_{2k}x^{2k} : k \in \mathbb{Z}, k \geq 0, \text{ and } a_i \in \mathbb{Z}\}$.
   Prove that S is a subring of R.

   (b) Let $\varphi : R \to \mathbb{Z} \times \mathbb{Z}$ be defined by $p(x) \mapsto (p(0), p(1))$. Show that φ is a ring homomorphism, and find $ker\varphi = \varphi^{-1}[\{(0,0)\}]$. (The kernel of a ring homomorphism will always be the preimage of the *additive* identity.)

UNGRADED HOMEWORK:

Starred problems from this list are classic results you will almost certainly need to use again. Ignore the external direct product business in TF.

Section	Problems
18	20, 22, 27, 28, 35, 37, 38, 49*