MATH 113, HOMEWORK #6
Due Thursday, October 1

Remember, consult the Homework Guidelines for general instructions. From now on, all work with permutations should be done in disjoint cycle notation. As before, any dihedral group problems should use our r, s notation.

GRADED HOMEWORK:

1. Mini prove or disprove problems. Determine whether each of the following statements is true (all separate). If so, give a (short) proof. If not, give a counterexample. Both proofs and counterexamples should be explained in complete sentences.
   (a) If G and H are groups such that G has exactly k different subgroups, and H has exactly ℓ different subgroups, then the group $G \times H$ has exactly $k \cdot \ell$ different subgroups.
   (b) Each dihedral group D_n (with $n \geq 3$) is isomorphic to the direct product of a group of order n and a group of order 2.
2. In the group D_6, let H be the subgroup $\{e, r^2, r^4\}$. (You should probably verify for yourself that H really is a subgroup, but you don't need to write this out.)
   (a) Using ideas from the proof of Lagrange's Theorem, we can see that G can be partitioned into four left cosets of H, or into four right cosets of H, with each coset containing 3 elements in either case. Compute the left and right cosets of H. Do the left cosets and the right cosets partition G in the same way? *(Name things sensibly. For example, one left coset will be $rH = \{r, r^3, r^5\}$. It could also have been called r^3H or r^5H, but rH looks cleanest.)*
   (b) Whenever a subgroup has the property that each left coset of H is also a right coset of H, it turns out that the cosets form a group. In particular, our four cosets above form a group of order 4. Write out the group table for this group. What familiar group is it isomorphic to? *(The process is similar to the example covered in Tables 10.5 and 10.6, but your cosets will have proper names like rH, not LT, MD, and DK.)*
   * *Think about, but don't turn in: this is actually really special behavior, to have the left and right cosets match. For an example where the left and right coset partitions are not the same, try the subgroup $\{e, s\}$ in D_6.*
3. Try to find an example of each of the following. Show that your example meets the criterion, or prove that no example is possible.
   (a) A subgroup of S_5 which has order 10.
   (b) An abelian subgroup of S_5 which has order 6.
   (c) A nonabelian group which has subgroups of order 3, 6, and 7 (and possibly others).
   (d) A subgroup of $GL(2, \mathbb{C})$ which has order 11.

UNGRADED HOMEWORK:

Starred problems from this list are classic results you will almost certainly need to use again. Ignore the external direct product business in TF.

Section	Problems
10	#19, 20, 21, 22, 23, 24, 28*, 29*
10	#30, 31, 32, 33, 34, 39, 40*, 41, 42
11	#14, 24, 25, 26, 29, 32, 36, 38, 50, 51*, 52