MATH 113, HOMEWORK #5
Due Thursday, September 24

Remember, consult the Homework Guidelines for general instructions. For any problems with permutations, write them in cycle notation. For dihedral groups, use the r, s notation from class.

GRADED HOMEWORK:

1. This problem deals with S_7 and U_{10}. You should probably use part (a) to do part (b).
   (a) Find an element of S_7 that has order 10. Call it x. List the elements of $G = \langle x \rangle$, the subgroup of S_7 generated by your element x (in cycle notation, not just $\{e, x, x^2, x^3, \ldots, x^9\}$). Find all generators of G, and make sure you explain why there are no others.
   (b) Determine the number of different isomorphisms $\phi : G \to U_{10}$. *(Hint: you might need a theorem from ungraded homework.)*
   * *Think about, but don't turn in: Can you find a homomorphism $\phi : G \to U_{10}$ which is not an isomorphism? What is its image? What does the preimage of a point in U_{10} look like?*
2. This problem deals with dihedral groups and symmetric groups. It may be helpful to use geometric intuition about symmetry groups when trying to think of examples.
   * *Think about, but don't turn in: Prove that if G and H are isomorphic groups (in general, not necessarily permutation groups), then they have the same number of elements of each possible order?*
   (a) The groups D_{12} and S_4 both have order 24. Prove that they are both nonabelian, but they are not isomorphic to each other.
   (b) Does D_{12} have a subgroup which is isomorphic to the Klein-4 group V? If so, find it and write out its group table. (You do not need to give an explicit isomorphism with V.) If not, explain why no such subgroup exists.
   (c) Find a subgroup of D_{12} which is isomorphic to S_3. Show that your answer is correct either by comparing the two group tables or by finding an explicit map which you show is an isomorphism.
3. Here's a definition (it's discussed a bit in Section 7, which you otherwise don't need to focus on). Suppose G is a group, and S is a subset of G. The *subgroup generated by S* is the smallest subgroup of G that contains all elements of S. For example, we have seen that the entire group D_n is generated by $\{r, s\}$. Of course, if S contains just a single element of G, then we are talking about cyclic subgroups, but we can sometimes make fancier subgroups when S has more elements.
   (a) In the group D_6, what is the subgroup L generated by $\{r^2, s\}$? What familiar group is L isomorphic to? Give a brief reason for the isomorphism, using geometric explanations or recent theorems (as opposed to our classic check by finding a map that is 1-1, onto, and satisfies the homomorphism property).
   (b) In S_{10}, what is the subgroup K generated by the two-element set $\{(1,8)(2,9), (3,7)(5,6)\}$? Is K a subgroup of A_{10}? Why or why not? What familiar group is K isomorphic to? Again, give a brief explanation.

UNGRADED HOMEWORK:

Section	Problems
8	#21, 22, 23, 24, 25, 26, 35, 44
9	#13*,23,29