MATH 113, HOMEWORK #2

Due Thursday, September 3

Remember, consult the Homework Guidelines for general instructions.

GRADED HOMEWORK:

1. Let V denote the set of vectors $\langle x, y, z\rangle$ in $\mathbb{R}^3$, with the operations of addition and subtraction. (Though vectors also have scalar multiplication, you can ignore that for this problem.) Consider the relation

   $\langle x_1, y_1, z_1\rangle \sim \langle x_2, y_2, z_2\rangle$ if and only if $x_1 - x_2 = y_1 - y_2 + k = z_1 - z_2 + 2k$ for some $k \in \mathbb{R}$.

   (a) Show that $\sim$ is an equivalence relation on V.

   (b) Describe the equivalence classes of $\sim$, giving a complete list. (Note: there are infinitely many equivalence classes, so you will need set-building notation or something similar.) Your explanation should make it clear that each vector of V belongs to **exactly** one equivalence class from your list; or in other words, your alleged equivalence classes do indeed partition V. Can you give both algebraic and geometric descriptions of the equivalence classes?

2. Use polar/exponential form for complex numbers in this problem ($re^{i\theta}$ form).

   (a) Write out the binary operation tables for $\mathbb{Z}_6 = \langle \mathbb{Z}_6, +_6\rangle$ and $U_6 = \langle U_6, \cdot\rangle$. Order the rows and columns in a sensible way.

   (b) Prove that the map $f : \mathbb{Z}_6 \to U_6$ given by $k \mapsto \left(e^{\frac{2\pi}{6}i}\right)^k$ is an isomorphism of binary structures.

   (c) Find a different isomorphism of binary structures, $g : \mathbb{Z}_6 \to U_6$, and prove your answer is an isomorphism.

3. Let $\phi : \mathbb{Z} \to \mathbb{Z}$ be defined by $\phi(n) = n - 3$ for all $n \in \mathbb{Z}$. For each part below, construct a binary relation $*$ (and note $*$ may be different in each part) so that ϕ is an isomorphism of binary structures. Justify your answers. Note: it is a trivial exercise to check that ϕ is $1-1$ and onto; you may assume this is true and just verify the homomorphism property in each case.

   (a) $\phi : \langle \mathbb{Z}, *\rangle \to \langle \mathbb{Z}, +\rangle$

   (b) $\phi : \langle \mathbb{Z}, +\rangle \to \langle \mathbb{Z}, *\rangle$

   (c) $\phi : \langle \mathbb{Z}, *\rangle \to \langle \mathbb{Z}, \cdot\rangle$

   (d) $\phi : \langle \mathbb{Z}, \cdot\rangle \to \langle \mathbb{Z}, *\rangle$

UNGRADED HOMEWORK:

Section	Problems
0	28, 36
1	35, 36, 37
2	24, 29, 30, 31, 32, 33, 34
3	11, 12, 13, 14, 15, 29, 33