

GIVE AN EXAMPLE PRACTICE

This handout is not for a grade; it's just practice problems for you to try. Some of these should be super easy, and some will be more challenging.

Give a *nontrivial* example of each of the following objects. All are possible.

1. A finite ring of matrices.
2. A ring which has exactly 16 elements and whose characteristic is 4.
3. A commutative subring of the noncommutative ring $M_n(\mathbb{Q})$.
4. A subring of \mathbb{C} which does not have unity.
5. A matrix ring which does not have unity.
6. Three units in $M_3(\mathbb{Z})$.
7. A ring homomorphism $\varphi : \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$.
8. A zero divisor of the ring $M_2(\mathbb{R})$.
9. A finite ring with at least 3 zero divisors.
10. A ring whose only units are 1 and -1 .
11. A solution of the equation $x^2 + 5x + 6$ in \mathbb{Z}_{12} which does not come from factoring.
12. A finite field with at least 20 elements.
13. An integer $k \geq 20$ such that 60 divides k^4 . (Hint: show both 5 and 12 are divisors.)
14. A zero divisor of the ring $\mathbb{Z}_5 \times \mathbb{Z}_7$.
15. A polynomial ring which is an integral domain.
16. A polynomial ring which is not an integral domain.
17. A ring without unity that has no zero divisors.
18. A ring in which the equation $(x + 2)(x - 5)(x - 3) = 0$ has exactly 3 different solutions.
19. An integer k such that $\varphi(k) = 8$, where φ denotes Euler's function.
20. An integral domain whose field of quotients is \mathbb{R} .