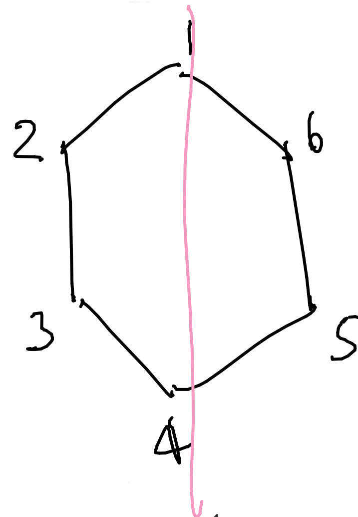
D_n Practice Handout - Math 113

(using D_6 as an example)

$D_6 = \langle \{$symmetries of  $\}$, composition$\rangle$



$r =$ 60° rotation CC

$s =$ reflection over vertical line

Rules:

$$r^6 = e$$
$$s^2 = e$$
$$rs = sr^{-1}$$

Using these, we can write every element in $s^a r^b$ form, where $a \in \{0, 1\}$, $b \in \{0, 1, 2, 3, 4, 5\}$

D_6 is definitely <u>not</u> <u>cyclic</u>, but it does have a generating set consisting of 2 elements, meaning every element of D_6 can be written as a finite product of these 2 elements. $\{r, s\}$ is the easiest generating set.

Remember to do symmetries R to L, since we are thinking of them as function composition.

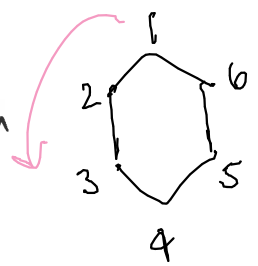
So $sr^2 = s \circ r^2$ = Rotate CC 120°
Then reflect over line through 1.

Note: our convention will be to let s stay fixed in the plane, not moving with vertex 1.

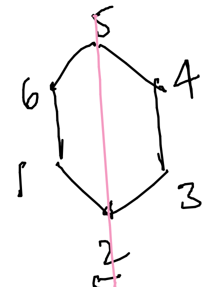
☆ fixed from original. ☆

So sr^2 in 2 steps:

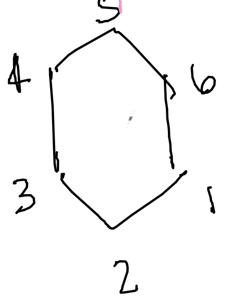
Start with:



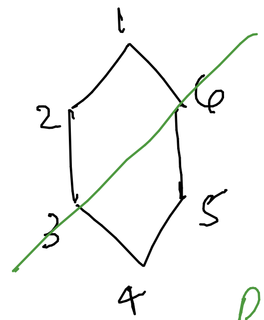
Do r^2:



Then s:



In one step:

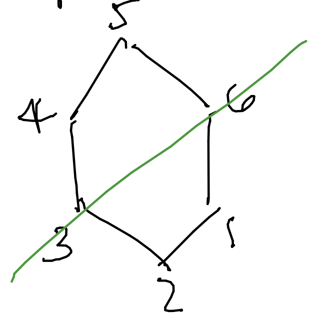


3,6 fixed

2,4 swap

1,5 swap

Reflect over green line.



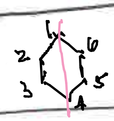
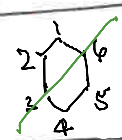
Practice: try filling in the rest of the table on the next page.

Also: practice with the rules. Simplify each of the following into $s^a r^b$ form using the rules D_n follows. (or anything else you learned by filling in the table.)

- r^{-3}
- $rsrs$
- $r^{-2} s r^{2}$
- $s^5 r^5 s r^2$
- $s r^{11} s$

Note: It should be straightforward to convince yourself (using geometry) that all the rules are true. the amazing part is that they completely determine D_n: every equation that is true in D_n can be built out of these facts.

D_6 Elements

Element	Geometric description	Order	Permutation of vertices (in cycle notation), $\{1, 2, 3, 4, 5, 6\}$
e	identity	1	e
r	rotate CC 60°	6	$(1, 2, 3, 4, 5, 6)$
r^2	rotate CC 120°	3	$(1, 3, 5)(2, 4, 6)$
r^3			
r^4			
r^5			
s	reflect over 	2	$(2 6)(3 5)$
sr			
sr^2	reflect over 	2	$(1 5)(2 4)$
sr^3			
sr^4			
sr^5			