MATH 104, HOMEWORK #5
Due Thursday, February 18

Remember, consult the Homework Guidelines for general instructions. Results from class, our textbook, and graded homework are fair game to use unless otherwise specified. You may also use ungraded homework results from previous problem sets.

GRADED HOMEWORK:

1. Let $t_1 = 1$ and $t_{n+1} = t_n\left(1 - \dfrac{1}{(n+1)^2}\right)$ for all $n \geq 1$.
   (a) Explain how the results in Chapter 10 guarantee that $\lim t_n$ exists, though they tell you nothing (or at most very little) about what the limit is.
   (b) Use induction to show that $t_n = \frac{n+1}{2n}$ for all $n \in \mathbb{N}$.
   (c) Find $\lim t_n$ and prove your answer is correct (either by quoting previous results or from scratch).

2. Let (a_n) be the sequence defined by $a_n = \frac{1}{n}$ if n is odd and $a_n = n$ if n is even. That is $(a_n) = (1, 2, \frac{1}{3}, 4, \frac{1}{5}, 6, \frac{1}{7}, 8, \ldots)$. Completely characterize (with proof) which subsequences of (a_n) have limits, and determine the set S of subsequential limits of (a_n).

3. Create a sequence (b_n) of positive real numbers whose set S of subsequential limits contains infinitely many numbers from the closed interval $[0, 1]$. You may clearly describe your sequence in words or with a recursive definition if you find it difficult to write an explicit formula. Prove that your answer meets the criterion.

* Extra Credit (3 points) – Find the serious mathematical error in the book's proof of Theorem 11.2 (i) and explain how to fix it. (There is a very quick fix.) Include a figure that shows what is going on. You do not need to rewrite the whole proof; just work on the portion that needs to be fixed.

UNGRADED HOMEWORK:

Pay special attention to starred problems; they are usually classics we will use many times, often important theorems hidden in the exercises.

Section	Exercises
10	3, 5, 10
11	1, 2, 3, 4, 8*, 9, 10, 11

* No really, make sure you know how to prove the Squeeze Theorem for sequences.