MATH 104, HOMEWORK #3

Due Thursday, February 4

Remember, consult the Homework Guidelines for general instructions.

GRADED HOMEWORK:

1. Give direct proofs for the two following limits. Do not use any of the shortcut theorems from Section 9 (e.g. limit of a sum, limit of a quotient).

   (a) Determine $\lim_{n\to\infty} \dfrac{3n^3+n^2}{4n^3-1001}$ and give a formal $\epsilon - N$ proof that your answer is correct.

   (b) Prove that $\lim_{n\to\infty} \dfrac{3n^3+n^2}{4n^2-1001} = +\infty$, using Definition 9.8 about sequences diverging to $\pm\infty$. (You might call this one an $M - N$ proof, though that is not a universal name.)

2. Prove the following, which is basically Exercise 9.12 in Ross (the book provides some hints).

   Let (a_n) be a sequence of real numbers such that $a_n \neq 0$ for all n. Assume that the limit $L = \lim_{n\to\infty} |\dfrac{a_{n+1}}{a_n}|$ exists (that is, it's a real number or $\pm\infty$).

   (a) Show that if $L < 1$, then $\lim_{n\to\infty} a_n = 0$.

   (b) Show that if $L > 1$, then $\lim_{n\to\infty} |a_n| = +\infty$.

   (c) *Think about, but don't turn in: find examples of sequences which show that the $L = 1$ case is inconclusive for this test, i.e (a_n) might converge or diverge.*

3. Let $t_1 = 1$ and for $n \geq 1$, let $t_{n+1} = \frac{(t_n)^2+6}{3t_n}$. Assuming that (t_n) converges, find $\lim_{n\to\infty} t_n$.

UNGRADED HOMEWORK:

Note that you should pay special attention to starred problems; they are usually classics we will use many times, often important theorems hidden in the exercises. There are a lot of these; some are just quick computations, and even with the proofs, you should not feel compelled to write all of them up beautifully, but you should figure out how they work. There are some hints and (partial) solutions for many odd problems at the end of the book. When practicing $\epsilon - N$ limit proofs, you may not need to do ALL of these, especially on multi-part problems; the point is to do enough that you are confident you can do most similar ones.

Section	Exercises
7	1, 2, 3, 4, 5
8	1, 2, 3, 4*, 5*, 7, 8, 9*
9	1, 2, 3, 6, 7, 8, 9*, 10, 11

PS: Ex. 8.5 called the Squeeze Theorem, not the Squeeze Lemma.