

Qualifying Exam Syllabus

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November 29th 2023, 9am - 12pm

Committee: Michael Lindsey (Exam Chair), Bernd Sturmfels (Advisor), Sylvie Corteel, David Eisenbud.

1 Major topic: Combinatorics (Algebra)

References: Richard Stanley, Enumerative Combinatorics, Chapters 1-3, 5, 7. Anders Claesson, Lecture notes on Combinatorial species, part 8-12, 15-16

- *Enumeration (Chapters 1-2,5):* inversions, descents, partitions, The Twelffold Way, stirling numbers, inclusion-exclusion, involutions, ordinary and exponential generating functions, Lagrange Inversion Formula, Matrix-tree theorem
- *Species (Lecture notes):* addition, multiplication, composition, derivative, pointing and Cayley's theorem
- *Posets and lattices (Chapter 3):* poset operations, distributive lattices, Fundamental Theorem of Distributive Lattices, chains and multichains, incidence algebra, zeta function, Möbius function, Möbius inversion
- *Symmetric Functions (Chapter 7):* partitions and their orderings, bases for the space of symmetric functions, Hall inner product, semi-standard Young Tableaux, Schur functions, Kostka numbers, Littlewood-Richardson rule, Jacobi-Trudi identity, Pieri rule

2 Major topic: Metric Algebraic Geometry (Applied Mathematics)

References: Paul Breiding, Kathlén Kohn and Bernd Sturmfels, Metric Algebraic Geometry (Most recent version: https://kathlenkohn.github.io/Papers/MFO_Seminar_MAG.pdf).

- *Euclidean distance optimization (Chapters 2-4):* critical equations, ED degree, Gröbner bases, parameter homotopy, homotopy continuation, polar varieties, polar degrees, Chern classes.
- *Metric invariants (Chapters 6-8):* curvature, tubular neighborhoods, medial axis, reach, bottlenecks, offset hypersurfaces, offset discriminant, Voronoi cells.
- *Algebraic statistics (Chapters 5, 10-11):* Wasserstein distance, polyhedral norms, independence models, machine learning, neural networks, convolutional networks, Kullback-Leibler divergence, maximum likelihood degree, Gaussian models.
- *Numerical aspects (Chapters 9, 12):* Condition numbers, Eckhart-Young Theorem, Distance to discriminant, tensor rank, eigenvectors and singular vectors, rank one varieties

3 Minor topic: Commutative Algebra (Algebra)

References: David Eisenbud, Commutative Algebra with a View Toward Algebraic Geometry, Chapters 2-4, 8-10, 13-15, 17.1-2, 18.1-2.

- *Rings and Modules (Chapters 2,4):* localization, tensor products, Nakayama's lemma, Hilbert's Nullstellensatz, Hilbert basis theorem.
- *Primary Decomposition (Chapter 3):* prime avoidance, associated primes, primary decomposition.
- *Dimension Theory (Chapters 8-10, 13-14, 17-18):* Krull dimension, Hilbert polynomials, Hilbert functions, Hilbert series, systems of parameters, principal ideal theorem, regular local rings, Cohen-Macaulay rings.
- *Gröbner bases (Chapter 15):* initial ideals, monomial ideals, Buchberger's algorithm, elimination, ideal membership, syzygies and free resolutions.