1 (a) Prove that the Newton iteration

\[ Df(x_k)(x_{k+1} - x_k) = f(s) - f(x_k) = -f(x_k) \]

converges quadratically to the solution \( s \) of \( f(s) = 0 \) whenever \( f : D \subset \mathbb{R}^n \rightarrow \mathbb{R}^n \) is differentiable on \( D \), the Jacobian matrix \( Df(x) \) is invertible for all \( x \in D \), there is exactly one \( s \in D \) with \( f(s) = 0 \), and \( Df \) satisfies the affine invariant Lipschitz condition

\[ \| Df(x)^{-1} (Df(x) - Df(y)) \| \leq L \| x - y \| \]

for all \( x, y \in D \).
(b) Write down explicit formulas for the Jacobian matrix $Df(u)$ and Newton update $v - u$ when $v = x_{k+1}$, $u = x_k$ and

$$f(x) = \begin{bmatrix} x_1^2 - x_2^2 - a \\ 2x_1x_2 - b \end{bmatrix}$$

for some positive numbers $a$ and $b$. 
(c) Find a domain $D$ containing the solution $s$ where the affine invariant Lipschitz condition is satisfied for the function $f$ of (b). (Hint: $\|s\|_2 = (a^2 + b^2)^{1/4}$.)

(d) Find a domain $G$ containing the solution $s$ such that Newton’s method is guaranteed to converge for the function $f$ of (b) from any starting point $x_0 \in D$. 