<table>
<thead>
<tr>
<th>Name</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td></td>
</tr>
<tr>
<td>1b</td>
<td></td>
</tr>
<tr>
<td>1c</td>
<td></td>
</tr>
<tr>
<td>2a</td>
<td></td>
</tr>
<tr>
<td>2b</td>
<td></td>
</tr>
<tr>
<td>3a</td>
<td></td>
</tr>
<tr>
<td>3b</td>
<td></td>
</tr>
<tr>
<td>3c</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>
(1a) Compute the complex Fourier coefficients of

\[ f(x) = \text{sign}(x) \]

on the interval \(-\pi < x < \pi\).
(1b) Let \( u(x, t) \) be the solution of the Schrödinger equation

\[
    u_t = iu_{xx},
\]

where \( i = \sqrt{-1} \), which is \( 2\pi \)-periodic in \( x \) and satisfies the initial condition \( u(x, 0) = f(x) \) from 1a. Find the complex Fourier coefficients \( \hat{u}(k, t) \) in terms of \( \hat{f}(k) \).
(1c) Show that

\[ u(x, 2\pi) = u(x, 0). \]
(2a) Suppose the discrete Fourier transform matrix $F$ is defined by

$$F_{jk} = \frac{1}{\sqrt{N}} e^{-2\pi i jk/N}$$

for $0 \leq j, k \leq N - 1$. Show that $F^4 = I$ is the identity matrix and deduce all possible values for the eigenvalues of $F$. 
(2b) Let
\[ f_j = e^{-i\alpha j} \]
where \( \alpha / 2\pi \) is an irrational number. Evaluate \( \hat{f} = Ff \).
(3a) Suppose $\varphi_n(x)$ is the orthonormal Hermite function

$$\varphi_n(x) = \frac{\pi^{-1/4}(-1)^n}{2^{n/2}\sqrt{n!}} e^{x^2/2} D^n e^{-x^2} = \frac{\pi^{-1/4}(-1)^n}{2^{n/2}\sqrt{n!}} \psi_n(x)$$

Show that

$$(D^2 - x^2)\psi_n(x) = -(2n + 1)\psi_n(x).$$
(3b) Suppose

\[ f(x) = \sum_{n=0}^{\infty} f_n \varphi_n(x) \]

where

\[ \sum_{n=0}^{\infty} |(2n + 1)f_n|^2 < \infty. \]

Evaluate the complex Fourier transform of \( f \).
(3c) Suppose
\[-iu_t(x, t) = (D^2 - x^2)u(x, t)\]
for $t > 0$ and $x \in \mathbb{R}$, and $u(x, 0) = f(x)$ from 3b. Find a value of $t$ such that
\[u(x, t) = g(t)\hat{f}(x)\]
is proportional to the complex Fourier transform $\hat{f}$ of $f$. 