Question 1  Fix $t > 0$ and let 

$$f(x) = \frac{e^{-x^2/4t}}{\sqrt{4\pi t}}.$$  

(a) Compute $\hat{f}(k)$.  
(b) Compute $\hat{f}(k)$ by a different method.

Question 2  Fix $A$ and $\alpha > 0$ and let $h(x) = Ae^{-\alpha x}$ for $x \geq 0$ and 0 otherwise.  
(a) Compute $\hat{h}(k)$.  
(b) Let  

$$f(x) = e^{-x} \left( \sin 5x + \sin 3x + \sin x + \sin 40x \right)$$  

for $0 \leq x \leq \pi$ and 0 otherwise. Compute $\hat{f}(k)$.  
(c) Plot $h \ast f(x)$ for $0 \leq x \leq \pi$ and find interesting values of $A$ and $\alpha$. Discuss.

Question 3  Fix $a > 0$ and let $f(x) = \sin^2 ax$ for $|x| \leq \pi$ and 0 otherwise.  
(a) Compute $\hat{f}(k)$.  
(a) Compute $\hat{f}(k)$ by a different method.  
(c) Explain the rate at which $|\hat{f}(k)|$ decreases as $|k| \to \infty$.

Question 4  Let $\varphi(x) = 1$ for $0 \leq x < 1$ and 0 otherwise. Show that  

$\varphi \ast \varphi(x) = 1 - |x - 1|$ if $0 \leq x < 2$ and 0 otherwise.