1. Compute an ON basis for the columns of
   \[ A = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix} = QR \]
   and find \( Q \) and \( R \).

2. Find \( a_0 \) and \( a_1 \) minimizing
   \[ F(a_0, a_1) = \int_0^1 (a_0 + a_1 x - e^x)^2 \, dx. \]

3. Find an ON basis for the 3-dimensional subspace of \( L^2(-1,1) \) generated by \( \{1, x, x^2\} \) and interpret as a QR factorization.
4. Let
\[ H_{n0} f(x) = \int_{-1}^{1} \frac{1}{x-y} f(y) \, dy \quad |x| \leq 1 \]
and
\[ H_{n1} f(x) = -\sum_{j=0}^{n} x^j \int_{-1}^{1} y^{j-1} f(y) \, dy \]
for \(|x| \leq 1\) and \(f \in L^2(-1,1)\).
Show that \(H_n\) is a bounded linear operator from \(L^2(-1,1)\) to \(L^2(-1,1)\) for \(0 \leq n \leq \infty\) and
\[ \|H_{n0} - H_{n1}\| \leq 2^{-n} \quad \text{for} \ n > 0. \]

5. Find an ON basis for the range of \(H_n\) and compute the orthogonal projection operator onto the range. Take \(n = 4\).