**True or False?**

Here is a list of true/false questions like those which may occur on the final. It will help you most if you learn not only *which* are true or false, but *why*. That way you’ll be ready for similar questions on the exam.

The list is weighted toward the end of the course. You should also go over the true/false sections on the midterms.

In the questions below, a *nice* scalar or vector field is one which has partial derivatives of all orders which are continuous everywhere.

1. A continuous function is always differentiable.
2. If the partial derivatives of $f$ exist and are continuous, then $f$ is differentiable.
3. $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 > 0\}$ is an open set.
4. The magnitude of the curl of a nice vector field $\mathbf{F}$ represents the amount of circulation per unit area in a plane normal to $\nabla \times \mathbf{F}$.
5. It is always true that $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{w}) = 0$.
6. For any nice scalar field $f$, $\nabla \cdot (\nabla f) \geq 0$ holds everywhere.
7. $\int_0^1 \int_0^y f(x, y)dx
dy = \int_0^1 \int_0^x f(x, y)dy
dx$ holds for all continuous functions $f(x, y)$.
8. The area of a region in the $xy$-plane enclosed by a piecewise-smooth, positively oriented, simple closed curve $C$ is given by $A = \int_C x
dy$.
9. If $S$ is a smooth surface, and $\mathbf{F}$ is a nice vector field that is orthogonal to the boundary of $S$ everywhere on the boundary of $S$, then it is true that $\iint_S (\nabla \times \mathbf{F}) \cdot dS = 0$.
10. If the height of a hill is described by a function $h(x, y)$, then a skier who wishes to descend the fastest should ski in a (compass) direction orthogonal to $\nabla h$.
11. $\nabla \times (\nabla \times \mathbf{F})$ is always equal to $\mathbf{0}$, for any nice $\mathbf{F}$.
12. If $S$ is a smooth oriented surface, and $-S$ is $S$ with the opposite orientation, then $\int_S f
dS = -\int_{-S} f
dS$ for any nice scalar field $f$.
13. If $C$ is an oriented smooth curve, and $-C$ is $C$ with the opposite orientation, then $\int_C \vec{F} \cdot d\vec{r} = -\int_{-C} \vec{F} \cdot d\vec{r}$ for any nice vector field $\vec{F}$.
14. For any nice scalar field $f$, $\text{div} (\text{grad}(f))$ is always 0.
15. If $S$ is a smooth oriented surface in $\mathbb{R}^3$, then $\iint_S \text{curl}(\vec{F}) \cdot d\vec{S} = 0$ for any nice vector field $\vec{F}$ in $\mathbb{R}^3$.
16. $\frac{d}{dt}(f(\vec{r}(t))) = \nabla f \cdot \frac{d\vec{r}}{dt}$ for any nice scalar field $f$ and smooth curve $\vec{r}(t)$. 
17. $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$, for any vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^3$.

18. If $f$ and $g$ are nice scalar fields on $\mathbb{R}^3$, and $\int \int \int_E f dV = \int \int \int_E g dV$ for every cube $E$, then $f = g$.

19. Let $C$ be a smooth curve in $\mathbb{R}^3$, and suppose $g$ is a nice scalar field such that $g(x, y, z) = k$ for all points $(x, y, z)$ on $C$. Then $\int_C \nabla g \cdot d\vec{r} = 0$.

20. If $\vec{F}$ and $\vec{G}$ are nice vector fields on $\mathbb{R}^3$ such that $\text{curl } \vec{F} = \text{curl } (\vec{G})$ at all points in $\mathbb{R}^3$, then $\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{G} \cdot d\vec{r}$ for any smooth, simple closed curve $C$.

21. If $\vec{F}$ and $\vec{G}$ are nice vector fields on $\mathbb{R}^3$ such that $\int_S \vec{F} \cdot d\vec{S} = \int_S \vec{G} \cdot d\vec{S}$ for any oriented smooth closed surface $S$, then $\text{div}(\vec{F}) = \text{div}(\vec{G})$ at all points in $\mathbb{R}^3$.

22. If $\vec{F}$ is nice, and $\text{curl}(\vec{F}) = \vec{0}$ at all points in $\mathbb{R}^3$, then $\int_C \vec{F} \cdot d\vec{r} = 0$, where $C$ is the arc of the circle $x^2 + y^2 = 1$ and $z = 0$, from $(1, 0, 0)$ to $(0, 1, 0)$.

23. For any vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2|\vec{a}|^2 + 2|\vec{b}|^2$.

24. Let $\vec{F}(x, y, z) = \frac{1}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}(x, y, z)$ and let $C$ and $D$ be oriented smooth curves in $\mathbb{R}^3 - \{(0, 0, 0)\}$ with the same initial and final points. Then $\int_C \vec{F} \cdot d\vec{r} = \int_D \vec{F} \cdot d\vec{r}$.

25. If $f_x(a, b)$ and $f_y(a, b)$ both exist, then $f$ is differentiable at $(a, b)$.

26. Let $\vec{F}(x, y, z) = (x, y, z)$, and let $S_1$ and $S_2$ be smooth closed surfaces enclosing simple regions of equal volume, oriented outward. Then $\int_{S_1} \vec{F} \cdot \vec{n} dS = \int_{S_2} \vec{F} \cdot \vec{n} dS$.

27. If $C$ is a smooth closed curve, and $\int_C \vec{F} \cdot d\vec{r} = 0$ then $\vec{F}$ is conservative.

28. If $f(x, y)$ is continuous, then

$$\int_{-\pi/2}^{\pi/2} \int_0^1 f(r \cos \theta, r \sin \theta) r dr d\theta = \int_{-1}^1 \int_0^{\sqrt{1-y^2}} f(x, y) dx dy.$$
30. \[
\int_0^\pi \int_0^\pi (x \cos(y^2) + y \sin(x^2)) \, dx \, dy \geq 2\pi^3
\]

31. If \( C \) is a circle of radius \( a \) centred at the origin, and \( f \) is a continuous function such that \( f(-x, y) = -f(x, y) \), then \( \int_C f(x, y) \, ds = 0 \).

32. If \( \mathbf{F}(x, y, z) = \langle P(x, y, z), Q(x, y, z), R(x, y, z) \rangle \) is a conservative vector field and \( P, Q, \) and \( R \) have continuous first partial derivatives then \( \frac{\partial P}{\partial x} = \frac{\partial R}{\partial z} \).

33. If \( f \) and \( g \) have continuous first partial derivatives and \( C \) is a smooth, closed curve then \( \int_C f \nabla g \cdot d\mathbf{r} = -\int_C g \nabla f \cdot d\mathbf{r} \).