

Math 136, Homework 11, due Thurs. April 23, 5pm

The post-midterm-2 material starts with the lecture notes dated 4/9/2009 (on logical consequence). This contains the proof of one of our major theorems, that validity in \mathcal{L}^{ENT} is not decidable.

After that, we'll go on to Chapter 9, as expanded by material I will cover in lectures and in notes I will hand out. We will look at ways of measuring complexity of predicates and functions which go beyond recursiveness and recursive enumerability. In particular, we'll introduce *Turing reducibility*, the *Turing jump operator*, and the *arithmetical hierarchy*.

I won't lecture on 9.1–9.3 because mostly we've already covered these ideas. The exercises from 9.1–9.3 below should be do-able now.

Exercises due April 23:

From the notes dated 4/9/2009 on logical consequence: exercise 9.

From chapter 9: page 165, problems 1–6.

Bonus problems.

- (6) Show that every existential sentence that is true in \mathcal{N}_0 is provable in A_E . (See hints in notes from 4/2/2009.)
- (7) Show that there are sets A and B of natural numbers such that neither is many-one reducible to the other, i.e. $\neg(A \leq_m B)$ and $\neg(B \leq_m A)$. [Hint: Construct the characteristic functions of A and B simultaneously. At step n you have finite initial segments s_n and t_n of these characteristic functions. Define s_{n+1} and t_{n+1} so as to defeat the n -th possible many-one reduction.]
- (8) Show that there are r.e. sets A and B such that neither is many-one reducible to the other. (This is much harder than problem 7. I will give hints to anyone interested in trying it.)