Math 136 Incompleteness and undecidability, Spring 2020

Text. Nigel Cutland, *Computability: an introduction to recursive function theory,* Cambridge University Press.

Instructor. John Steel, 717 Evans, coremodel@berkeley.edu

Office Hours. TTh 2:30–4.

Class meeting times. 12:30-2:00 TTh, 310 Hearst Mining.

Prerequisites Math 55 would help. Math 125A might help a little. Math 113, or some equivalent experience with abstract mathematics (definitions, theorems, and proofs), might help. Nevertheless, the course will be self-contained.

Course Plan The course title sounds a bit negative, doesn't it? Actually, we'll cover some of the most beautiful theorems in Logic, results of Church, Turing, Kleene, and Godel from the 1930's. These theorems established basic limitations on what can be computed by algorithm, and what is provable in axiomatic systems. Among these results are Kurt Gödel's famous incompleteness theorems, which we will cover toward the end of the semester.

There is some overlap between this couse and CS 170, but it is not extensive. It amounts to about 4 weeks of CS 170, the part where the basic general notion of computable-by-algorithm is introduced. In contrast to CS 170, we shall not go in the direction of time-bounded or space-bounded "practical" computability. Instead, we shall consider computability in a more general logical context.

We shall follow Cutland's book, supplemented with handouts later in the course. There is no pre-requisite for the course other than a general readiness to take upper division math. This will indeed be a rigorous math course, stressing exact definitions and honest proofs. It might be helpful to have taken 125A, or even just 110 or 113, but it is not necessary.

We shall proceed at roughly the rate of one chapter per week. Since there are 12 chapters, that gives us some leeway. If we don't need to use the leeway to catch up, I will use it to expand chapters 6 and 8 with lecture

material. I have a detailed set of lecture notes from the first time this course was given, and will distribute them to the class.

In several places, Cutland gives proof sketches for major results, rather than real proofs. His sketches are quite good, but I will use some of the extra time to expand them.

Exams and homework. Homework will be assigned weekly. The assignments will announced at lecture, and posted on the web at

`http://math.berkeley.edu/~steel/courses/Courses.html`

They will also be posted at the Bcourses site for this course. Homework solutions will be posted there as well.

There will be two midterms, the first in late February after we have covered chapter 5, and the second in April. I will announce the exact date for each midterm at least 2 weeks in advance of it.

Homework will be due weekly. In general, it will be due the week following its being announced. It will be collected via Gradescope. Please ask if you are not familiar with Gradescope.

There will be a written final exam as well.

Grading. Each midterm is worth 20%, and the final exam is worth 40%. Homework and other coursework accounts for the remaining 20%.

TA. The GSI for this course is Benjamin Siskind. His office is 937 Evans, and his office hours are MWF 11-12:30.

There's nothing like working through some problems with someone providing hints or explanations and elaborations as you go. For that reason, I encourage you to come to my office hour, or Siskind's (or both!).