Suggested Reading Materials to Prepare for
MATH 110 Linear Algebra

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Part I: General Preparation for Math Upper-Division Courses at UCB:

On Proofs and the Logic of Communicating Mathematics

If you can, we highly recommend taking concurrently

- **MATH 74 Transition to Upper-Division Mathematics**
  (a.k.a. Introduction to Proofs)

Otherwise, **study** the following chapters in **"A Decade of the Berkeley Math Circle, the American Experience" vol. I**, AMS/MSRI Mathematical Circles Library:

Session 5: A Few Words about Proofs. I

1. Why Prove Things?
2. Proofs versus Non-Proofs
3. Proof by Contradiction
4. Proofs of Possibility and Impossibility
5. Some Problems Need Two Proofs!
6. Hints and Solutions

Session 6: Mathematical Induction

1. Examples and Conjectures
2. Mathematical Induction and Proof
3. Mathematical Induction in Action
4. Strong Induction
5. Mathematical Induction in Other Areas
6. A Word of Caution
7. Hints and Solutions

Session 9: Complex Numbers

(Optional, but highly recommended!
Read as much as you can.)

1. A Problem from Geometry
2. Some History
3. Complex Numbers via Geometry
4. Basic Operations on Complex Numbers
5. Complex Multiplication
6. Another Form of Complex Numbers
7. Summary: What Have We Learned?
8. Hints and Solutions

Watch the following **Numberphile videos** (search "Numberphile Stankova"):

1. **Pebbling a Chessboard:** Learn about invariants, proof by contradiction, prove beyond a reasonable doubt that something is impossible, and incorporate geometric series into game theory!
2. **Conway's Checkers:** Learn about monovariants, strengthen your proof-by-contradiction techniques, discover the golden ratio and its "evil" twin and use them to solve another game mystery!
3. **The Three-Square Geometry Problem:** Incorporate 5^{th} grader's knowledge into a puzzle with at least 54 different solutions, some of which probably going back to a famous tangram by Archimedes.
4. **Triangles have a Magic Highway:** Get a sip of high school geometry that you always wanted to have but never had a chance to, along with Euler's famous line (of course, Euler!)
5. **A Miraculous Proof (Ptolemy's Theorem):** Learn about inversion in the plane, who is "stronger" – Pythagoras or Ptolemy – and how the golden ratio makes it again in a famous polygon you have learned to draw from an early age, but have you looked deep enough into it?

Part II: Specific Preparation for Math 110 Linear Algebra

Topics to Review from Previous Linear Algebra Courses

"*Linear Algebra with Applications,"* Otto Bretscher, Prentice Hall
(or any other comparable introductory textbook to Linear Algebra)

1. **Systems of Linear Equations:**
    a. Matrix form of systems
    b. Row reduced echelon form (RREF)
    c. Solving systems and the space of their solutions
2. **Matrix Algebra**
    a. What are matrices? Operations on matrices
    b. Properties of matrices and operations on them
    c. Finding the inverse of a square matrix using RREF
3. **Determinants**
    a. What are determinants? Properties of determinants
    b. Direct formulas for determinants of 2x2 and 3x3 matrices
    c. Direct formula for the Inverse of 2x2 matrices
    d. Recursive formula for the determinant of n x n matrices using Laplace expansion along a row or column
    e. Cramer's rule for solving systems of linear equations, using determinants
4. **Vector Spaces $\mathbf{R}^n$ and Their Subspaces**
    a. What are the vector spaces $\mathbf{R}^n$? What are vector subspaces of $\mathbf{R}^n$?
    b. Properties of vector subspaces of $\mathbf{R}^n$
    c. Linearly independent and spanning sets. Bases and dimensions
    d. Coordinates in $\mathbf{R}^n$ and their subspaces
5. **Linear Transformations between spaces $\mathbf{R}^n$**
    a. What are linear transformation between spaces $\mathbf{R}^n$?
    b. Properties of linear transformations
    c. Image (range) and kernel (null space) of linear transformations
    d. Rank and nullity of a linear transformations
    e. Type of linear transformations: 1-1, onto, and invertible
    f. Matrix of a linear transformation
    g. Inverse of a linear transformation
    h. Dimension Theorem for linear transformations
6. **Dot Product in $\mathbf{R}^n$**
    a. What is the dot product in $\mathbf{R}^n$?
    b. Properties of the dot product
    c. Applications of the dot product