1. Given are \( n+1 \) real linear equations in \( n \) variables (of the form \( a_1x_1 + a_2x_2 + \cdots + a_nx_n = a \)). Prove that each \( = \) sign can be replaced with either \( \leq \) or \( \geq \) so that the resulting \( n+1 \) inequalities have the following property: for every choice of real numbers \( x_1, x_2, \ldots, x_n \), at least one inequality is true.

2. Let \( m, n \) be positive integers. Suppose that a given rectangle can be tiled (without overlaps) by a combination of horizontal \( 1 \times m \) strips and vertical \( n \times 1 \) strips. Show that it can be tiled using just one of the two types.

3. The set \( S \) is a finite subset of \([0, 1]\) with the following property: for all \( x \in S \), there exist \( a, b \in S \cup \{0, 1\} \) with \( a, b \neq x \) such that \( x = (a + b)/2 \). Prove that all the numbers in \( S \) are rational.

4. Certain cities are connected by roads connecting pairs of them. The roads intersect only at the cities. A subset of the roads is called \textit{important} if destroying those roads would make it so that there are two cities such that it is impossible to go from the first to the second. A subset \( S \) of the roads is called \textit{strategic} if it is important and if no proper subset of \( S \) is important. Let \( S \) and \( T \) be distinct strategic sets of roads. Let \( U \) be the set of roads that are in either \( S \) or \( T \), but not both. Prove that \( U \) is important.

5. Let \( ABCDEF \) be inscribed in circle \( k \) so that \( AB = CD = EF \). Assume that diagonals \( AD, BE, CF \) intersect in point \( Q \), and let \( P = AD \cap CE \). Prove \( CP/PE = AC^2/CE^2 \).