Review Exercises for Midterm 2, Calculus 1A
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1. Graph the Original Function from the Graph of its Derivative

Problem 1. The graph of the derivative $f'(x)$ of a function $f(x)$ is in #31, p.305.
(a) Where is the original function $f(x)$ increasing and where is it decreasing? Explain.
(b) Where does $f(x)$ have local minima and maxima? Explain.
(c) Where is $f(x)$ concave up and where is it concave down? Explain.
(d) Where does $f(x)$ have inflection points? Explain.
(e) What happens at $x = 6$? Which of the functions $f(x)$, $f'(x)$ and $f''(x)$ is not defined there? Explain.
(f) Suppose $f(0) = 2$. Sketch a possible graph of the original function $f(x)$.

2. Graph the Original Function from its Algebraic Formula

Problem 2. Consider the function $f(x) = \frac{x^2 + 4}{x}$.
(a) Find all roots of $f(x)$ (if any). Explain.
(b) Find all vertical, horizontal and slant asymptotes of $f(x)$ (if any). Explain.
(c) Find all intervals where $f(x)$ is increasing and where it is decreasing. Explain.
(d) Find all local maxima and minima of $f(x)$. Explain.
(e) Find all intervals where $f(x)$ is concave up and where it is concave down. Explain. Does $f(x)$ have inflection points? Why?
(f) Sketch a graph of $f(x)$, including all features you found above.

3. Apply MVT or Rolle’s Theorem

Problem 3. Does there exist a function $f(x)$ such that $f(0) \leq -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all $x$?

4. Apply L’Hospital’s Rule

Problem 4. Find the following limit. Justify your solution.
(a) $\lim_{x \to 1^+} (x - 1) \cdot \tan \frac{\pi x}{2}$
(b) $\lim_{x \to 1} \left( \frac{1}{\ln x} - \frac{1}{(x-1)} \right)$

5. Apply Implicit Differentiation

Problem 5. Find all points on the curve $x^2y^2 + xy = 2$ where the tangent line slope is $-1$.

6. Solve an Optimization Problem

Problem 6. The top and the bottom margins of a poster are each 6 cm, and the side margins are each 4 cm. If the area of the printed material on the poster is fixed at 384 cm$^2$, find the dimensions of the poster with smallest area.

7. Bonus Problems

Problem 7*. The frame for a kite is to be made from six pieces of wood. The four exterior pieces have been cut with the lengths 3 and 4. To maximize the area of the kite, how long should the diagonals be? (Hint: set $x$ to be half of the horizontal diagonal in the textbook picture.)

Problem 8*. In an automobile race along a straight road, car $A$ passed car $B$ twice. Prove that at some time during the race their accelerations were equal. (Hint: Apply Rolle’s Theorem.)