Problem 1. Consider the graph of \( f(t) \) in #2, p.402. Let \( g(x) = \int_0^x f(t) \, dt \).

(a) On what intervals is \( g(x) \) increasing? Why?
(b) On what intervals is \( g(x) \) concave up? Why?
(c) Calculate the 7th midpoint approximation \( M_7 \) of \( g(7) \).
(d) Calculate the exact value of \( g(5) \).

Problem 2. Let \( y(x) = \int_{x^2 - \frac{\pi^2}{4} + 1}^{\tan x} \frac{1}{\sqrt{2 + t^4}} \, dt \). Find

(a) \( \lim_{x \to \frac{\pi}{4}} y(x) \). Explain.
(b) \( y'(\frac{\pi}{4}) \). Explain.

Problem 3. A body moves along a straight line with acceleration \( a(t) = 2t + 3 \, m/sec^2 \). The velocity at time \( t = 0 \) is \( v(0) = -4 \, m/sec \).

(a) Find the velocity and the speed functions for the time period \(-3 \leq t \leq 10\). Draw their graphs.
(b) Find the average speed for the time period \([-3 \, sec, 10 \, sec]\).
(c) Find the displacement of the body at time \( t = 10 \, sec \) relative to the position at \( t = -3 \, sec \).
(d) Find the total distance travelled for the time interval \([-3 \, sec, 10 \, sec]\).

Problem 4. Consider the following integrals:

\[
A = \int_1^4 2x \ln x \, dx, \quad B = \int_0^3 2x \ln(x + 1) \, dx, \quad C = \int_1^4 2(x - 1) \ln x \, dx, \quad D = \int_0^9 \ln(\sqrt{x} + 1) \, dx.
\]

Which of these integrals are equal to each other? Explain. (Substitution Rule will be helpful here.)

Problem 5. Find the numbers \( a \) such that the average value of the function \( f(x) = (x^2 + 3) \) on the interval \([a, 1]\) equals 4. Find all \( c \in [a, 1] \) for which \( f(c) = 4 \).

Problem 6. Let \( f(x) = x^4 + 1 \) and \( g(x) = \sqrt[4]{x} + 1 \).

(a) Draw the region \( R \) determined by the two curves \( f(x) \) and \( g(x) \), and bound on the left by \( x = 0 \) and on the right by \( x = 2 \).
(b) Find the area of the region \( R \).
(c) Set up a formula for the volume of the solid defined by rotating the region \( R \) about the \( x \)-axis.
(d) Set up a formula for the volume of the solid defined by rotating the region \( R \) about \( x = -3 \).

Problem 7. Calculate \( \int \tan x \ln(\cos x) \, dx \). (Use Substitution twice.)

Problem 8. If \( f'(x) \) is continuous on \([a, b]\), show that \( 2 \int_a^b f(x)f'(x) \, dx = [f(b)]^2 - [f(a)]^2 \). (Hint: Substitute.)

Problem 9. Calculate the volume of a solid whose base is the region between the curves \( y = x^2 \) and \( y = 1 \), and whose cross-sections perpendicular to the \( y \)-axis are squares.

Problem 10. Find \( \lim_{h \to 0} \frac{1}{h} \int_2^{2+h} \sqrt{1 + t^3} \, dt \). (Use LH and FTC.)