Undecidability of the α-degrees

Theodore A. Slaman
(on joint work with Chong Chi Tat)

University of California, Berkeley

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Overview

- Interpreting models within degree structures
- Techniques and their applications for the Turing degrees
- Joy and woe of trying the same for the α-degrees

Interpretations of First Order Structures

Let $\mathcal{M}$ and $\mathcal{N}$ be first order structures with finite signature.

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  - With the property that $\mathcal{N} \xrightarrow{\sim} (N/\equiv, c_j^{\mathcal{M}}, f_j^{\mathcal{M}}, R_j^{\mathcal{M}})$.

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- There there is a translation from the common theory of the models in this family to $Th(\mathcal{M})$.

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Corollary (Follows from earlier results of Lachlan)

The theory of the Turing degrees is undecidable.

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To reverse the comparison and interpret second order arithmetic in $\mathcal{D}$, we need to interpret second order quantifiers over a countable interpreted model.

Theorem (Slaman and Woodin)

Every countable relation on $\mathcal{D}$ is uniformly definable from finitely many parameters in $\mathcal{D}$.

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- A uniform method with which to interpret set quantifiers over a countable coded model.

Theorem (Simpson)

The first order theory of $\mathcal{D}$ is recursively isomorphic to the second order theory of arithmetic.

Earlier proofs by Simpson and Nerode-Shore used interpretations of first order arithmetic by initial segments and Spector's exact pair theorem to interpret set quantifiers.

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If we can define a reasonable definability-neighbor of a within $\mathcal{D}$, then we can interpret a countable set of reals which includes a representative of a.

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Theorem (Jockusch and Shore)

The set of degrees arithmetic in $\mathbf{a}$ is definable from $\mathbf{a}$, using the analysis of bases for cones of minimal covers.

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Proof

The reals interpreted using parameters arithmetic in $\mathcal{O}$ and above $\mathcal{O}$ contains an element which is not arithmetic.

The Bi-interpretability Conjecture

Conjecture (Slaman and Woodin)

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Theorem (Slaman and Woodin)

- *The Bi-interpretability Conjecture is true relative to parameters.*
- *It is equivalent to $\mathcal{D}$'s being rigid.*
- *It implies that a relation is definable in $\mathcal{D}$ iff it is induced by a degree-invariant relation definable in second order arithmetic.*

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The Turing jump is definable in $\mathcal{D}$.

Recently, Shore has given a localized proof of the definability of the jump.

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An ordinal α is Σ_1-admissible iff L_α satisfies Σ_1-replacement and Δ_1-comprehension.

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Examples of admissible ordinals:

- ω
- $\omega_1, \omega_2, \ldots, \omega_\omega, \ldots$ any cardinal
- ω_1^{CK}, the supremum of the recursive ordinals

α-Recursion Theory (Sacks)

Let L_α be fixed and consider definability within L_α.

Definition

- An α-reduction is a Σ_1 (in parameters) subset Φ of L_α consisting of quadruples (P, N, P^*, N^*).
- For A and B contained in α, $\Phi(A) = B$ iff for every disjoint P and N in L_α,

$$(P \subseteq B \,\&\, N \cap B = \emptyset) \iff \exists (P^*, N^*)[P^* \subseteq A \,\&\, N^* \cap A = \emptyset \,\&\, (P, N, P^*, N^*) \in \Phi]$$

- $A \geq_\alpha B$ iff there is an α-reduction Φ such that $\Phi(A) = B$.

α-Recursion Theory

deconstructing recursion theory

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For example, to separate and clarify the roles of these concepts – eliminate the role confusion and role strain that they endure in the standard setting.

α-Recursion Theory

immediate differences

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Consider Spector's construction of a minimal degree. A single step is determined by the Boolean value of a Π_2 condition. Σ_1-admissibility is not sufficient to show that the construction produces a set of minimal degree.

In the absence of a solution to the minimal degree problem, interpreting structures using initial segments of the α-degrees is impossible. Gone are the interpretations of Lachlan, Simpson, and Nerode-Shore. Gone, too, is the analysis of cones of minimal covers.

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Relative to sets A such that α is not Σ_1-admissible relative to A, the analogies between recursive and α-recursive are not helpful. Reveal an uncountable singularity in α and all the similarities between the α-degrees and the Turing degrees are gone.

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undecidability

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We modify the apparatus of the Slaman-Woodin coding theorem for the Turing degrees.

α-Recursion Theory

defining anti-chains from parameters

In both the Turing degrees and the α-degrees, we define an anti-chain $\mathcal{A} = \{A_1, \ldots, A_n\}$ from parameters as follows.

- Let B be the join of $\mathcal{A}$.
- Find C_1 and C_2 such that the following conditions hold.
  - For all $A_i \in \mathcal{A}$, there is a G such that $G \not\leq A_i$, $G \leq A_i + C_1$, and $G \leq A_i + C_2$, i.e. $C_1 + A_i \wedge C_2 + A_i \neq A_i$.
  - For all $W \leq B$, either there is an $A_i \in \mathcal{A}$ such that $W \geq A_i$ or $C_1 + W \wedge C_2 + W = W$.

Defining Anti-Chains from Parameters

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general α

We begin by assuming that each A_i is the set of initial segments of a regular and hyper-regular set W_i. That is, for all $\beta < \alpha$, $W_i \bigcap \beta \in L_\alpha$, and $L_\alpha[W_i]$ is Σ_1-admissible.

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Explicit use of regularity replaces the original implicit use of finiteness.

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Analysis of the forcing relation and effective approximation replace the ω-length recursion to build generic sets.

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    - *Is there an interpretation of the second order theory of L_α within the α-degrees?*
  - *Is there an automorphism of the α-degrees?*

Finis