SINGULARITIES AND LONG TERM DYNAMICS OF THE SELF-DUAL CHERN-SIMONS-SCHRÖDINGER EQUATION

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The self-dual Chern-Simons-Schrödinger equation (CSS) is a gauged version of the 2D cubic nonlinear Schrödinger equation (NLS) introduced by the physicists Jackiw and Pi in 1990. While initially noted for the special 'self-dual' structure that allows for finding explicit(!) static solutions (i.e., solitons), recent studies have revealed remarkable dynamical features of this model that are either different or not known for other related models, such as the 2D cubic NLS, Schrödinger maps, wave maps, etc. The goal of this lecture series is to discuss such features for equivariant self-dual CSS – including soliton resolution with no bubble tree, the construction and classification of blow-up/long term dynamics etc. – and to discuss some (possible) implications for other nonlinear dispersive models.

The plan for each lecture is as follows.

1. Lecture I: Soliton resolution with no bubble tree

In this lecture, I will introduce the self-dual Chern-Simons-Schrödinger equation (CSS) in equivariance symmetry and discuss some of its basic properties, such as the existence of solitons and pseudoconformal blow-up solutions. The lecture will culminate with the proof of *soliton resolution* for this model, following [6].

2. Lecture II: Rotational instability mechanism and backward blow-up construction

In this lecture, we will first construct a one-parameter family of solutions that exhibits the so-called *rotational instability mechanism* for the pseudoconformal blow-up solutions. We will then discuss the main ideas of a backward blow-up construction in [4] (see also [1, 7]), which exhibits a large family of blow-up solutions with the pseudoconformal blow-up rate and a similar rotational instability mechanism.

3. Lecture III: Forward blow-up dynamics

In this lecture, we will discuss the forward-in-time blow-up dynamics for CSS. References for this lecture include [3, 5, 2].

References

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4. ______, *On pseudoconformal blow-up solutions to the self-dual Chern-Simons-Schrödinger equation: existence, uniqueness, and instability*, Mem. Amer. Math. Soc. **284** (2023), no. 1409, vi+128. MR 4574850
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