LONG TIME BEHAVIOR OF WAVES ON ASYMPTOTICALLY FLAT SPACETIMES: UNIFORM DECAY AND LATE TIME TAIL

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The goal of this lecture series is to give a coherent exposition of classical and recent techniques for studying the late time behavior of solutions to wave equations, which are possibly highly nonlinear, on asymptotically flat spacetime. The plan for each lecture is as follows.

1. Lecture I: Vector field method and uniform decay

The *vector field method* is a powerful tool for establishing the uniform decay of solutions to linear and nonlinear wave equations. First introduced by Klainerman [2] in the 1980's, the method has been further extended by many authors. In this lecture, particular emphasis will be given to the importance of *integrated local energy decay* (or *Morawetz estimate*), which is a weak but robust form of decay, and the method of r^p*-weighted energy* introduced by Dafermos–Rodnianski [1] and further developed by Schlue [5] and Moschidis [4].

2. Lecture II & III: Late time asymptotics

In these lectures, we will start from the vector field bounds established in Lecture I and upgrade these into the understanding of the precise leading order late time asymptotics (or tails) when the space dimension d is odd and ≥ 3. In the linear stationary case, we will see that there are many possible cancellations leading to rather fast late time decay – this is connected to the *strong Huygens principle* for waves on $\mathbb{R}^{1+d}$ with $d \geq 3$ odd, as well as *Price's law* in black hole physics. On the other hand, we will also see that such fast decay rates are in general unstable with respect to nonlinear and/or nonstationary perturbations of the equation. The reference for this lecture is [3].

References

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2. Sergiu Klainerman, *Uniform decay estimates and the Lorentz invariance of the classical wave equation*, Comm. Pure Appl. Math. **38** (1985), no. 3, 321–332. MR 784477
3. Jonathan Luk and Sung-Jin Oh, *Late time tail of waves on dynamic asymptotically flat spacetimes of odd space dimensions*, (2024).
4. Georgios Moschidis, *The r^p-weighted energy method of Dafermos and Rodnianski in general asymptotically flat spacetimes and applications*, Ann. PDE **2** (2016), no. 1, Art. 6, 194. MR 3493208
5. Volker Schlue, *Decay of linear waves on higher-dimensional Schwarzschild black holes*, Anal. PDE **6** (2013), no. 3, 515–600. MR 3080190

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