ON LONG TIME DYNAMICS FOR THE SELF-DUAL CHERN–SIMONS–SCHRÖDINGER EQUATION WITHIN EQUIVARIANT SYMMETRY: RIGIDITY OF BLOW-UP AND ROTATIONAL INSTABILITY

KIHYUN KIM

In this lecture series, we will consider the classification of long time dynamics of solutions, to so called *self-dual Chern–Simons–Schrödinger equation* (CSS), under equivariant symmetry (a variant of radial symmetry, roughly speaking). This equation was introduced by the physicists Jackiw and Pi in the 90s [1], as a gauged (or covariant) 2D cubic nonlinear Schrödinger equation (NLS) with an additional structure called *self-duality*. Although it looks more complicated than (NLS) at first glance, this geometric nonlinear Schrödinger equation turns out to have a surprisingly rigid long time dynamics (at least within equivariant symmetry): (i) soliton resolution, (ii) rigidity of finite-time blow-up, and (iii) rotational instability. We will explore this rigid dynamics using *modulation analysis*.

In the first lecture, we will focus on soliton resolution for (CSS) in $H^{1,1}$. The main reference will be [3]. Soliton resolution roughly says that under generic assumptions all solutions decompose into the sum of finitely many modulated (i.e., scaled and phase rotated depending on time) solitons and a radiation. Some distinguished features of (CSS) will allow a much stronger statement of soliton resolution for (CSS) to hold (namely, at most one soliton can arise in the resolution) with a much shorter proof. We will try to present the proof as much as possible.

In the second and third lectures, building upon the soliton resolution result, we will proceed further towards the classification of the asymptotic behavior of scale and phase (of the modulated soliton). The main reference will be [2]. This problem will require a more refined analysis including the study of linearized dynamics around soliton, the derivation of a universal ODE system governing the singular dynamics (*modulation equations*), the construction of modified profiles, and so on. As a result, we will observe the rigidity of smooth finite-time blow-up; all H^3 finite-time blow-up solutions (with high equivariance) must contract in space with constant speed (i.e., the scale $\sim (T-t)$ as $t \to T$ with the blow-up time $T < +\infty$) and their phases converge. Moreover, at least formally, we will observe that they are unstable through the mechanism called *rotational instability*.

References

[1] R. Jackiw and S.-Y. Pi. Soliton solutions to the gauged nonlinear Schrödinger equation on the plane. *Phys. Rev. Lett.*, 64(25):2969–2972, 1990.

[2] K. Kim. Rigidity of smooth finite-time blow-up for equivariant self-dual Chern-Simons-Schrödinger equation. *preprint arXiv:2210.05412, to appear in J. Eur. Math. Soc.*, 2022.

[3] K. Kim, S. Kwon, and S.-J. Oh. Soliton resolution for equivariant self-dual Chern-Simons-Schrödinger equation in weighted Sobolev class. *preprint arXiv:2202.07314, to appear in Amer. J. Math.*, 2022.

Email address: `khyun@ihes.fr`

IHES, 35 route de Chartres, 91440 Bures-sur-Yvette, France