Mini-course: Nonlinear Hyperbolic PDEs

(under MAT290)

Lecturer: Sung-Jin Oh

(sjoh@math.berkeley.edu)

Monday and Wednesday, 4:10–5:30 p.m., 939 Evans

The aim of this mini-course is to provide an introduction to nonlinear hyperbolic PDEs, with an emphasis on the *vector field method* as a key tool for understanding the long time behavior of solutions. We will illustrate this method with several applications, including a proof of nonlinear global stability of the Minkowski space, which is a landmark result in mathematical general relativity.

The vector field method refers to a set of physical space techniques (as opposed to those relying on, say, Fourier or spectral analyses) that seeks to capture the long time dispersive behavior of solutions in a robust fashion. As the name suggests, the method revolves around the use of well-chosen vector fields on the background space-time, which are often related to exact or approximate symmetries of the equation. It has proven to be tremendously powerful in the study of highly nonlinear hyperbolic PDEs, such as Einstein's gravitational field equations in general relativity, and continues to play an indispensable role in recent breakthroughs, including dynamic formation of black holes, development of shocks in compressible fluids and analysis on black hole backgrounds.

A highlight of the mini-course will be presentation of a proof of global stability of the Minkowsi space for the vacuum Einstein equations of general relativity. This theorem was first established in the monumental work of Christodoulou-Klainerman (1993), using a tensor-geometric version of the vector field method. Here we will instead follow a more recent approach due to Lindblad-Rodnianski (2010), which is significantly shorter and requires less machinery.

Our plan is to start from the very basics, beginning with the linear wave equation on $\mathbb{R}^{1+d}$, and build our way up to the Lindblad-Rodnianski proof. No prior knowledge of PDEs or general relativity will be assumed.

First meeting: Jan 26, 2015