

1. Show that among every five integers, two have the same remainder when divided by 4.

2. Show that when 25 girls and 25 boys are seated around a circular table, ~~both~~ there exists a person both of whose neighbors are boys.

3. Show that in a group of 10 people, either three all know each other or four all do not know each other.

4. Show $\binom{2n}{2} = 2\binom{n}{2} + n^2$

5. Show $k\binom{n}{k} = n\binom{n-1}{k-1}$.