Recall that the Fibonacci sequence is defined by $F_1 = 1$, $F_2 = 1$, $F_n = F_{n-1} + F_{n-2}$.

Prove:

1. $\sum_{i=1}^{n} F_i = F_{n+2} - 1$ for all $n \in \mathbb{N}$.

2. For any positive integer m, F_{3m-2} and F_{3m-1} are odd, and F_{3m} even.

3. Prove that every positive integer n has a unique representation $F_{a_1} + \dots + F_{a_k} = n$ where $a_1, \dots, a_k$ are nonadjacent numbers.

4. Prove that $f_1^2 + f_2^2 + \dots + f_n^2 = f_n f_{n+1}$ for $n \in \mathbb{N}$.

5. Prove that $f_{n+1} f_{n-1} - f_n^2 = (-1)^n$ for $n \in \mathbb{N}$.