MATH 55 WORKSHEET 07/11/17

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1. Prove using induction that $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n. What is the base case and the inductive step?
2. Prove using induction that $n! < n^n$ when n is an integer greater than 1.
3. Let $H_n = \frac{1}{1} + \frac{1}{2} + ... + \frac{1}{n}$ denote the nth harmonic number. Prove that $H_1 + ... + H_n = (n+1)H_n - n$.
4. Prove that a set with n elements has $\frac{n(n-1)}{2}$ subsets of size 2 whenever $n \geq 2$.
5. Show that if n is a positive integer, then

$$\sum_{\{a_1,...,a_k\} \subset \{1,...,n\}} \frac{1}{a_1 a_2 ... a_k} = n.$$

(Note that this sum is over all nonempty subsets of the set $\{1, ..., n\}$.)