

1. Find $7^{121} \bmod 13$.

2. Find $3^{216} \bmod 130$. (Hint: $130 = 2 \cdot 5 \cdot 13$.)

3. An integer $a \neq 0$ is a quadratic residue of p if $x^2 \equiv a \pmod{p}$ has a solution.
What are the quadratic residues of 11?

4. Show that if $p \nmid a$, $x^2 \equiv a \pmod{p}$ has either no solution or two distinct solutions mod p .

5. Show that if p is an odd prime, there are exactly $\frac{p-1}{2}$ quadratic residues among the integers $1, 2, \dots, p-1$.

6. Show that a is a quadratic residue ^{mod p} iff $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$.