

1. Find ^{the} integer a such that

a) $a \equiv -15 \pmod{27}$ and $-26 \leq a \leq 0$

b) $a \equiv 24 \pmod{31}$ and $-15 \leq a \leq 15$

c) $a \equiv 99 \pmod{41}$ and $100 \leq a \leq 140$.

2. Convert

a) $(11111)_2$

b) $(10000001)_2$

c) $(10101001)_2$

to decimal.

3. Show that a positive integer is divisible by 3 iff the sum of its decimal digits is divisible by 3.

4. Find the prime factorization of

a) 39 b) 91 ~~91~~ c) 108.

5. Call a positive integer perfect if it equals the sum of its positive divisors other than itself.

a) Show that 6 and 28 are perfect.

b) Show that $2^{p-1}(2^p - 1)$ is perfect when $2^p - 1$ is prime.

6. Show that if $2^n - 1$ is prime, then n is prime.