Math 198 **Name:**

Problem 1.

Recall our universal property of group homomorphisms.

Theorem 1. *Given two groups G, H and a group homomorphism $f : G \to H$, let K be a normal subgroup of G such that $K \subseteq \ker f$, and let $\varphi : G \to G/K$ be the natural map identifying an element $g \in G$ with its coset $(g) \in G/K$.*

Then there exists a unique homomorphism $h : G/K \to H$ such that $f = h \circ \varphi$.

Now recall from Math 113 our notion of abelianization.

Definition 1. *Let G be a group. Then, the commutator subgroup of G is a normal subgroup $\{ghg^{-1}h^{-1} \mid g, h \in G\}$. The* ***abelianization*** *of G is defined to be the quotient group of G by the commutator subgroup, i.e.*

$$G^{ab} := G/\{ghg^{-1}h^{-1} \mid g, h \in G\}.$$

Prove the following assertion: Given a group G, an abelian group A, and a group homomorphism $f : G \to H$, let $\varphi : G \to G^{\mathrm{ab}}$ denote the natural map to the abelianization of G (a quotient of G).

Then there exists a unique homomorphism $h : G^{\mathrm{ab}} \to A$ such that $f = h \circ \varphi$.

Problem 1.

Problem 2.

Recall from Math 113 the (finitary) statement of Cayley's Theorem.

Theorem 2. *Let G be a finite group. Then there exists $n \in \mathbb{N}$ such that $H \subset S_n$ and $G \cong H$.*

If G has order k and elements of orders $e_1, \ldots, e_k$, then what might be some possible values of n in the theorem statement? Or can nothing be said – and if not, what might help determine n?