Math 198 **Name:**

Problem 1.

For any prime p and positive integer n, there exists a unique field (up to isomorphism) with p^n elements.

(a) Consider field $\mathbb{F}_2$. $\mathbb{F}_2$ has a field extension with 4 elements given by a root of a quadratic polynomial. Find this polynomial, and describe what the extension looks like. Can you further extend this field to a field of 8 or 16 elements?

(b) Consider field $\mathbb{F}_p$ where p is an odd prime (choose a specific one, if you like). Demonstrate a field extension $\mathbb{F}_p \subsetneq K, K' \subsetneq L$ where $K \neq K'$ each have p^2 elements and L has p^3 elements. Demonstrate an isomorphism between K and K'.