Math 198 **Name:**

Problem 1.

Let A be a commutative ring with 1, and let $\mathcal{C}$ denote the category of A-algebras, i.e.

- Objects of $\mathcal{C}$ are commutative rings B together with a ring homomorphism $A \to B$.
- Morphisms of $\mathcal{C}$ between A-algebras B and C are ring homomorphisms $B \to C$ that commute with the structure maps $A \to B$, $A \to C$.

Find objects to represent the following functors $\mathcal{C} \to \mathbf{Set}$.

(a) $B \mapsto B^\flat$

(b) $B \mapsto (B^\flat)^n$ for $n \in \mathbb{N}$

(c) $B \mapsto B^\times$, the set of units of B

(d) $B \mapsto \mathrm{Gl}_n(B)$, the set of invertible linear transformations of B^n

Problem 1.

Problem 2.

Let **Ab** be the category of abelian groups, whose morphisms are group homomorphisms. Then consider the "inclusion" functor $\iota : \mathbf{Ab} \to \mathbf{Grp}$ that maps both objects and morphisms to themselves. Prove that ι has a left adjoint. Draw a few pictures (diagrams) to help explain your point.