Math 55, Probability Worksheet

March 17, 2014

1. What is the probability that a positive integer less than or equal to 100 selected at random is divisible by either 5 or 7?

2. Which is more likely, getting a sum of 9 from rolling 2 dice, or getting a sum of 9 from rolling 3 dice?

3. Let's say you place 8 rooks randomly on a chess board (so that they all sit in different squares). What is the probability that they will not be able to attack each other (i.e. no two will be in the same row or column)?

4. Imagine you roll a pair of dice 24 times in a row. Let $P((m, n))$ for $1 \leq m, n \leq 6$ denote the probability that the pair of integers (m, n) are rolled at least once in those 24 times.

   - Guess whether $P((6, 6)) > 1/2$, $P((6, 6)) < 1/2$, or $P((6, 6)) = 1/2$. Don't think too hard.

- Now compute $P((6,6))$ and see if your guess was right

- True or false: the probability of rolling double of any integer (i.e. rolling $(1,1),(2,2),(3,3),(4,4),(5,5)$, or $(6,6)$) at least once during 24 dice rolls is $6 \cdot P(6,6)$?

5. What if there were four doors in the Monty Hall puzzle, and the host still opened one empty door after your choice? What is the probability of winning if you stick to your original choice, and what is the probability of winning if you switch?

6. Imagine that you want to adopt a cat from a pet shelter. There are n cats that need homes, and they are ordered randomly. You only have room in your apartment to adopt one cat. The way this shelter works is as follows: you sit in a room and the first cat is led in. You have five minutes to play with it and then you have to decide on the spot if you want to reject it or adopt it. If you adopt it, then you go home right then without seeing the rest of the cats. If you reject it, then you reject it forever and can no longer adopt it. You continue this until you adopt a cat (or if you still haven't decided at the end then you must adopt the last cat you see). Assume that if you were able to see all the cats at once, you would be able within five minutes to order them unambiguously from 1 to n, 1 being your favorite and n being your least favorite. Thus each time you see a new cat, you can rank it in terms of all the cats you've seen before (but you have no idea about how the cats you haven't seen yet compare). What is the optimal strategy if your goal is to maximize the probability of adopting your favorite cat?