Problem 1. Show that the matrix

\[
A = \begin{pmatrix}
-1 & -1 & 2 & 1 \\
-3 & 0 & 3 & 1 \\
-3 & -2 & 5 & 1 \\
1 & -2 & 0 & 0
\end{pmatrix}
\]

can not be brought to its Jordan canonical form over \( \mathbb{R} \) and find its rational canonical form.

Problem 2. Find the exponent \( e^A \) of the matrix

\[
A = \begin{pmatrix}
-2 & 2 & 1 & 3 \\
1 & -1 & -1 & 0 \\
-1 & 2 & 0 & 3 \\
-1 & 3 & 1 & 2
\end{pmatrix}
\]

Problem 3. Let \( \mathbb{R}[x] \) and \( \mathbb{R}(x) \) be respectively the ring of polynomials and the field of rational functions of \( x \) with real coefficients. For any \( a(x), b(x) \in \mathbb{R}[x] \), show that

\[
\mathbb{R}[x]/(a(x)) \otimes_{\mathbb{R}[x]} \mathbb{R}[x]/(b(x)) \simeq \mathbb{R}[x]/(d(x))
\]

where \( d(x) \) is the greatest common divisor of \( a(x) \) and \( b(x) \), and that

\[
\mathbb{R}[x]/(a(x)) \otimes_{\mathbb{R}[x]} \mathbb{R}(x) \simeq 0.
\]

Problem 4. Find all irreducible polynomials of degree 2 over \( \mathbb{F}_2 \), and use it to show that

\( p(x) = x^4 + x^3 + x^2 + x + 1 \) is also irreducible over \( \mathbb{F}_2 \). Let \( \theta \) be the root of \( p(x) \) in the field \( K = \mathbb{F}_2[x]/(p(x)) \), find \( \theta^{-1} \in K \).

Problem 5. Prove that a field with 8 elements cannot be an extension of a field with 4 elements.