

Math 53: Tu-Thurs, 8-9:30AM

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Office Hours:

Right after class, outside

Course Website:

www.math.berkeley.edu/~sethian/course.html

Grade Calculation:

30% First midterm

30% Second Midterm

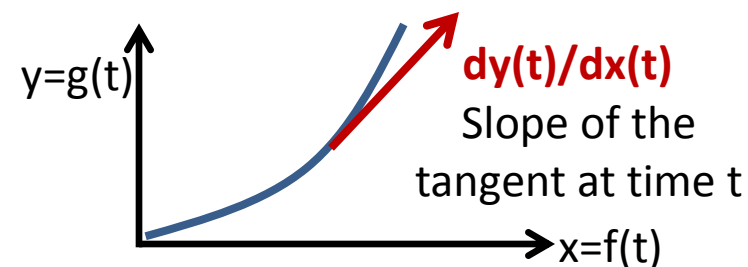
30% Final Exam

10% Homework and quizzes

Review of 10.1 and 10.2: (what we did last time)

We talked about parameterized curves

- $x = f(t)$ $y = g(t)$
- Example: $x(t) = 3t$; $y(t) = 8t^3$
- In order to write y as a function of x , we used the chain rule:



$$\begin{array}{c}
 t \quad \xrightarrow{\quad} \quad \boxed{} \quad \xrightarrow{\quad} \quad x \quad \xrightarrow{\quad} \quad \boxed{} \quad \xrightarrow{\quad} \quad y \\
 \text{Chain Rule: } \frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} \quad \xrightarrow{\quad} \quad \frac{dy}{dt} / \frac{dx}{dt} = \frac{dy}{dx} \\
 \text{(assume } dx/dt \neq 0\text{)}
 \end{array}$$

So now we can compute the slope of tangent lines:

Example: Suppose $x(t) = \sqrt{t}$ and $y(t) = t^2 - 2t$ Find the tangent line at $t=4$

Solution: Step 1: $dy/dx = [dy/dt]/[dx/dt] = \frac{2t-2}{(1/2)t^{-1/2}} = 4(t-1)\sqrt{t}$ At $t = 4$ we get $\frac{dy}{dx} = 24$

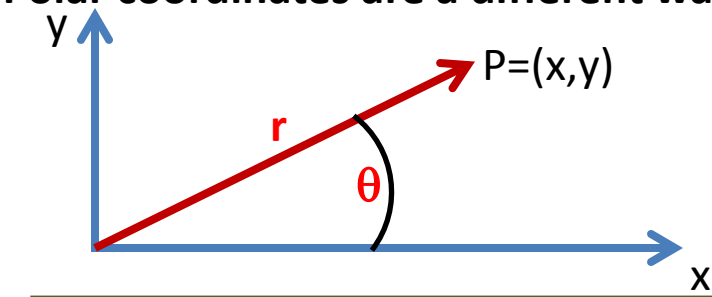
Step 2: Find tangent point and tangent line

At $t=4$, point is $(2,8)$, so tangent line is $(y-y_0) = \text{slope} * (x-x_0)$
 $(y-8)=24*(x-2)$

Section 10.3

Polar coordinates are a different way to characterize $\mathbb{R}^2$

- [$\mathbb{R}^2$ is a fancy way to write 2D]
- [$\mathbb{R}^1$ is the real line]
- [$\mathbb{R}^3$ is 3D space]

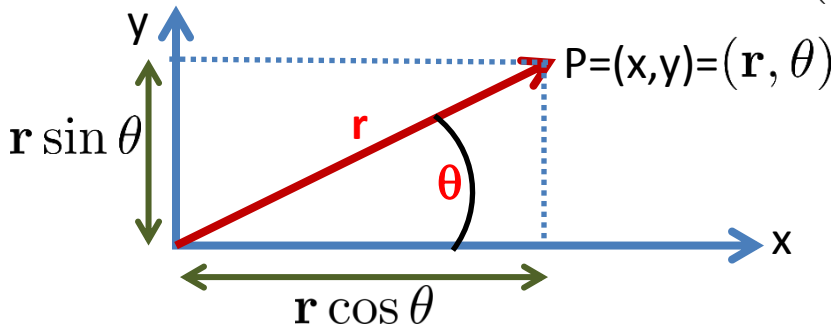


Examples of polar (r, θ) representations of points:

$(5, 0), (5, 2\pi), (5, 4\pi), (-5, \pi)$

All the same point (just to make this less crazy, we'll rule out $r < 0$)

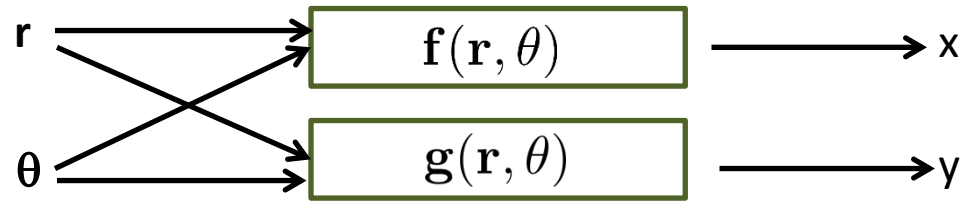
Question: How do we go from polar (r, θ) coordinates to Cartesian (x,y) coordinates?



So:

$$x = f(r, \theta) = r \cos \theta$$
$$y = g(r, \theta) = r \sin \theta$$

It is useful to start thinking about function boxes

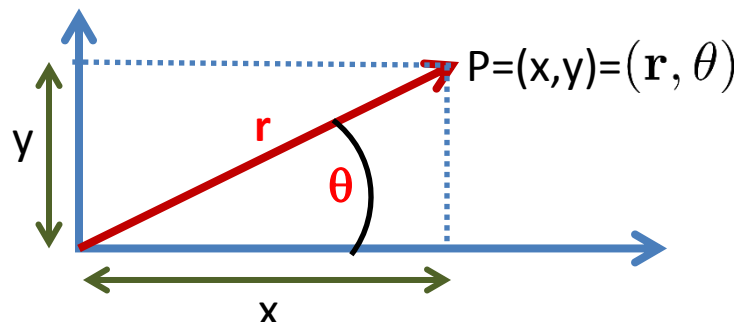


Start to think about mappings: this is a mapping from $\mathbb{R}^+ \times \mathbb{R}^1 \rightarrow \mathbb{R}^1 \times \mathbb{R}^1 = \mathbb{R}^2$

(actually, I should write $[0, \infty)$ instead of $\mathbb{R}^+$ but I'm too lazy to figure out how to do it...)

Section 10.3

Question: How do we go from Cartesian (x,y) coordinates to polar (r, θ) coordinates?



$$r = \sqrt{x^2 + y^2}$$

So:

$$\tan \theta = \frac{y}{x}$$

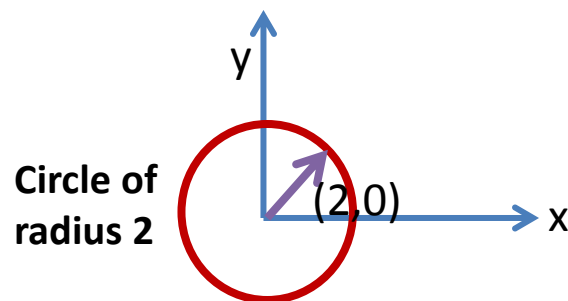
Let's get fancy with our notation: this is a mapping $\mathbb{R}^2 \rightarrow \mathbb{R}^+ \times \mathbb{R}^1$

Graphing in polar coordinates:

(Cranky
editorial
comments:)

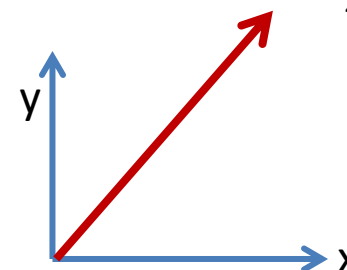
- (1) *This is non-intuitive.*
- (2) *It's a pain in the neck.*
- (3) *But you should see it once in your life.*
- (4) *And today's the day.*

Question: What does the curve $r=2$ look like?



Question: What does the curve $\theta = \frac{\pi}{4}$ look like?

Straight
line

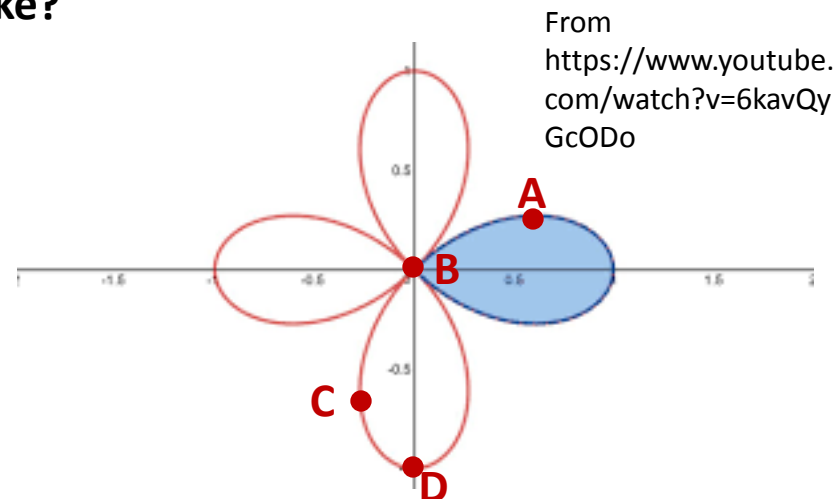


Section 10.3

Question: What does the curve $r = \cos(2\theta)$ look like?

Solution: First, try plotting some points:

Point	θ	r	x	y
A	$\frac{\pi}{8}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} \cos \frac{\pi}{8}$	$\frac{\sqrt{2}}{2} \sin \frac{\pi}{8}$
B	$\frac{\pi}{4}$	0	0	0
C	$\frac{3\pi}{8}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2} \cos \frac{3\pi}{8}$	$-\frac{\sqrt{2}}{2} \sin \frac{3\pi}{8}$
D	$\frac{\pi}{2}$	-1	0	-1



Suppose we are given a polar curve $r = r(\theta)$. How do we find tangents to polar curves?

Step 1: We have

$$x = r \cos \theta = r(\theta) \cos \theta$$

$$y = r \sin \theta = r(\theta) \sin \theta$$

So we are viewing θ as parameterizing the curve

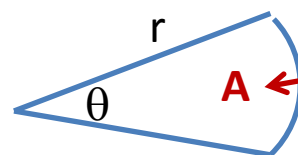
Step 2: So now we can find dy/dx :

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta} \sin \theta + r(\theta) \cos \theta}{\frac{dr}{d\theta} \cos \theta - r(\theta) \sin \theta}$$

Section 10.4

Question: How can we find areas in polar coordinates?

Recall that:



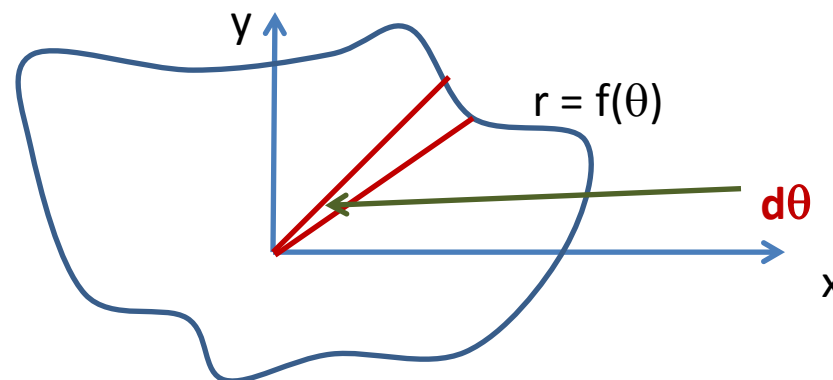
$$\text{Area} = (1/2) r^2 \theta$$

Why?

So we integrate all the slivers and find an area integral:

Consider a curve $r = f(\theta)$

$$\begin{aligned} \text{Area of little sliver} &= (1/2) [f(\theta)]^2 d\theta \\ &= (1/2) r^2 d\theta \end{aligned}$$

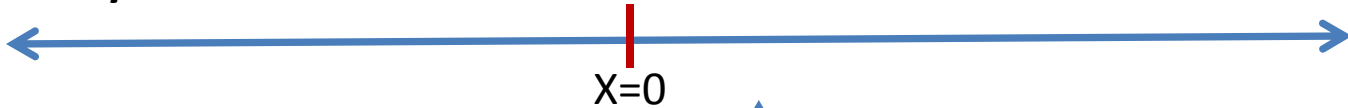


So it's now calculus—add them all up:

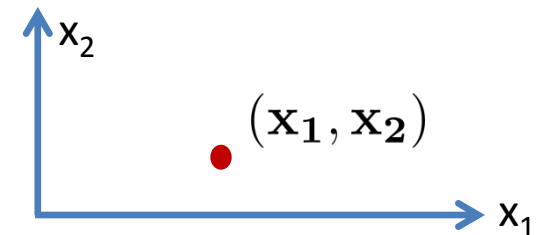
$$\text{Area} = \int_{\theta=\alpha}^{\theta=\beta} \frac{1}{2} r^2 d\theta$$

Section 12.1

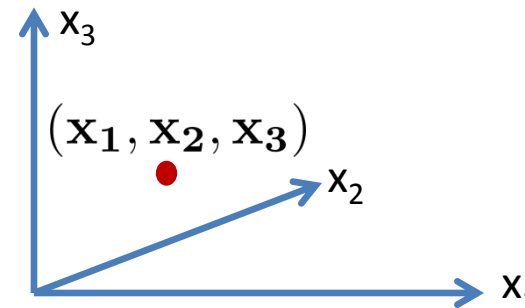
And now the course actually starts!

$\mathbb{R}^1$ The real line: 

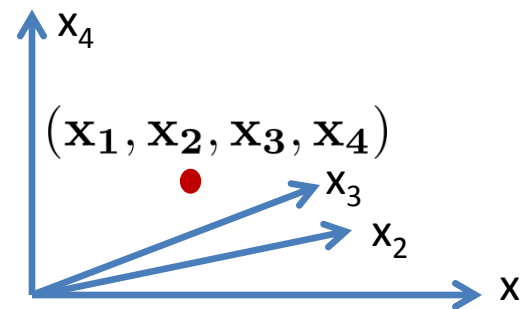
$\mathbb{R}^2$ Two-dimensional space = the plane



$\mathbb{R}^3$ Three-dimensional space



Why stop there? We have $\mathbb{R}^4$



Just because I can't
draw it doesn't mean
it's not there!

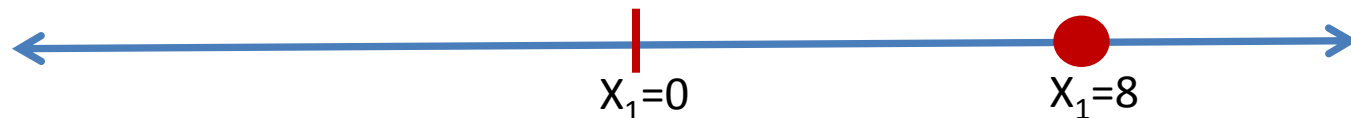
$\mathbb{R}^5$? (sure!) $\mathbb{R}^6$? (why not?)

Section 12.1

Some examples:

(1) Question: what is the object in $\mathbb{R}^1$ represented by $x_1=8$?

Answer:



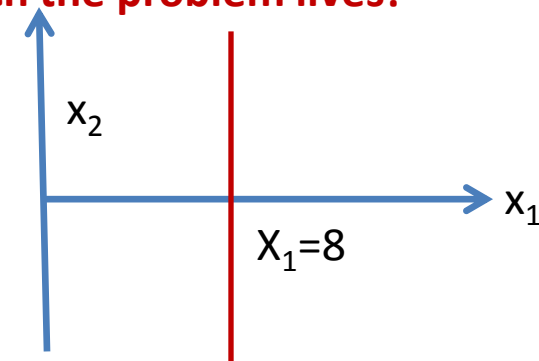
It's a point located at $x_1=8$ that divides one-dimensional space into two parts

Note that the answer depends on the dimension in which the problem lives!

(2) Question: what is the object in $\mathbb{R}^2$ represented by $x_1=8$?

Answer:

It's the line located at $x_1=8$ that divides two-dimensional space into two parts

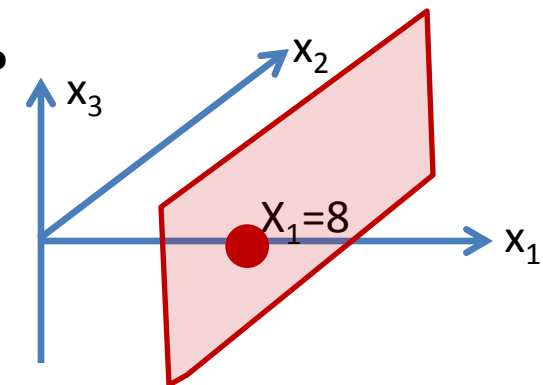


Note that the answer depends on the dimension in which the problem lives!

(3) Question: what is the object in $\mathbb{R}^3$ represented by $x_1=8$?

Answer:

It's the plane located at $x_1=8$ that divides three-dimensional space into two parts

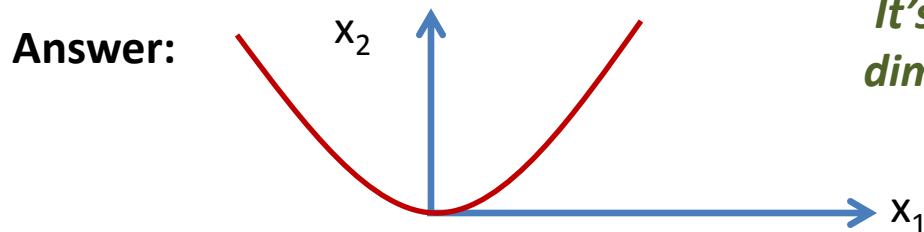


Note that the answer depends on the dimension in which the problem lives!

Section 12.1

More examples:

(3) Question: what is the object in $\mathbb{R}^2$ represented by $x_2 = x_1^2$?



It's a parabola that divides two-dimensional space into two parts

(4) Question: what is the object in $\mathbb{R}^4$ represented by $x_1 = 8$?

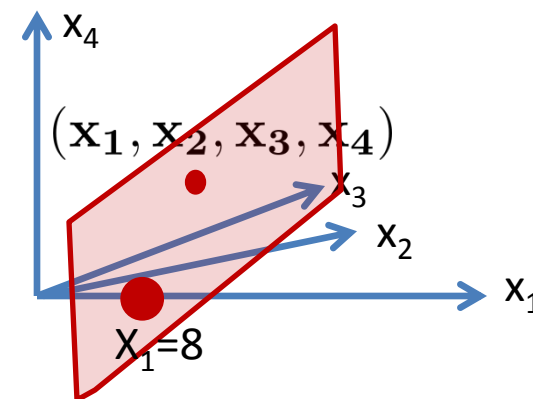
Answer: Well, I can't draw it---but the answer is all "4-tuples"

$$(x_1, x_2, x_3, x_4)$$

in 4D space such that $x_1 = 8$

It divides 4D space into 2 parts—
each of which is a 4D space

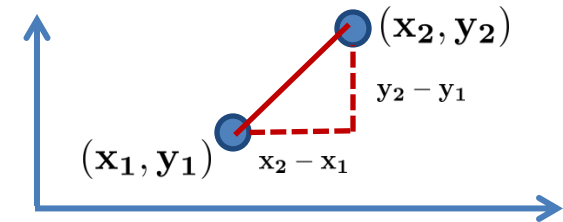
(you gotta open up your mind to see this!)



Section 12.1

What is the distance between two points

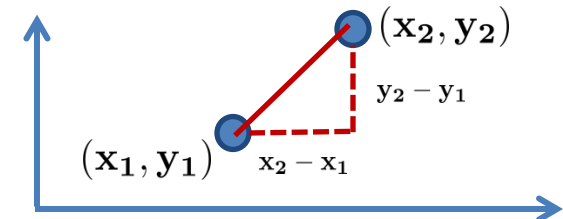
$(x_1, y_1), (x_2, y_2)$ in $\mathbb{R}^2$?



$$\text{Pythagorean Theorem} = \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

What is the distance between two points

$(x_1, y_1, z_1), (x_2, y_2, z_2)$ in $\mathbb{R}^3$?



$$\text{Pythagorean Theorem} = \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

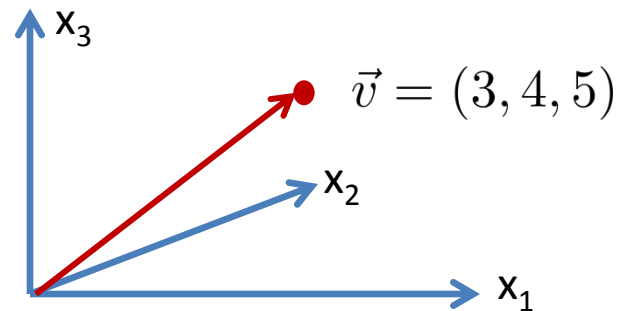
How can we use this to find equation for a sphere of radius R in $\mathbb{R}^3$ centered around (j,k,l) ?

Answer: Want to write an expression for all (x,y,z) that a distance R from the point (j,k,l)

$$R = \sqrt{(x - j)^2 + (y - k)^2 + (z - l)^2}$$

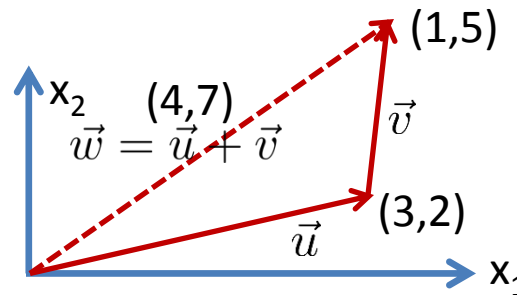
Section 12.2

Vectors:



Vector addition: $\vec{u} = (3, 2)$ $\vec{v} = (1, 5) \rightarrow \vec{u} + \vec{v} = (3 + 1, 2 + 5) = (4, 7)$

Geometrically:

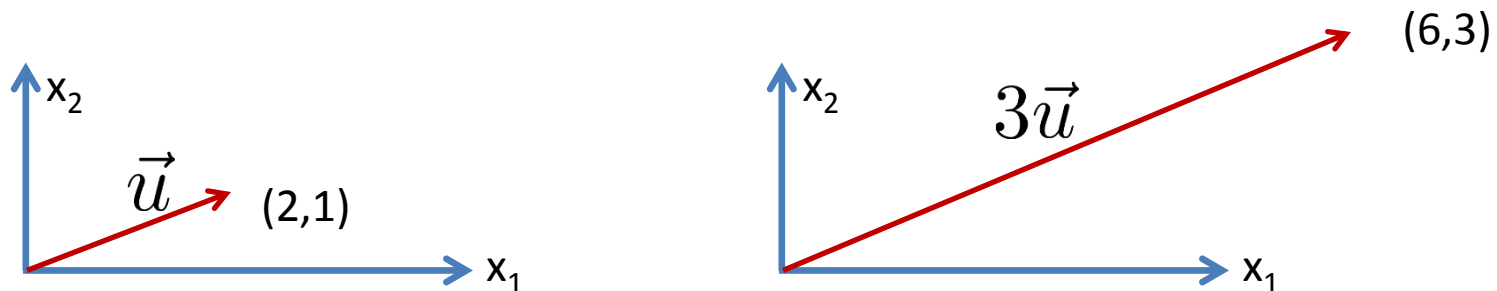


Section 12.2

We can do lots of things with vectors....

Thing 1: Definition: Scalar Multiplication

If c is a scalar, and $\vec{v}$ is a vector, then the scalar multiple $c\vec{v}$ is defined as a new vector that points in the same direction as $\vec{v}$ with length c times the original length



Section 12.2

We can do lots of things with vectors....

Thing 2: Definition: Length of a vector

$\vec{v} = (v_1, v_2, v_3)$ then the length of $\vec{v}$ is given by $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

Thing 3: Properties of Vectors:

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
- $\vec{a} + \vec{0} = \vec{a}$
- $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$
- $cd(\vec{a}) = c(d\vec{a})$
- $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
- $\vec{a} + (-\vec{a}) = \vec{0}$
- $(c + d)\vec{a} = c\vec{a} + d\vec{a}$
- $1\vec{a} = \vec{a}$