Seismic velocity estimation and time to depth conversion of time-migrated images

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SUMMARY

We address the problem of time to depth conversion of time migrated seismic images and show that the Dix velocities estimated from time migration velocities are the true seismic velocities divided by the geometrical spreading of image rays. We pose an inverse problem: to find seismic velocities from Dix velocities and suggest two algorithms for solving this problem. One algorithm is based on the ray tracing approach, and the second is based on the level set approach. We test these algorithms on synthetic data examples and apply them to a real data example from the North Sea. We demonstrate that it is important to take into account the fact that, in laterally heterogeneous media, the Dix velocities are not equal to the seismic velocities and that the difference between the two can be significant.

INTRODUCTION

Time-domain seismic imaging is a robust and efficient process routinely applied to seismic data (Yilmaz, 2001). Rapid scanning and determination of seismic velocities during time migration can be accomplished, for example, by velocity continuation (Fomel, 2003). Time migration is considered adequate for seismic imaging in areas with mild lateral velocity variations. However, even mild variations can cause structural distortions of time-migrated images and render them inadequate for accurate interpretation of subsurface structures.

To remove structural errors inherent in time migration, it is necessary to convert time-migrated images into the depth domain either by migrating the original data with a prestack depth migration algorithm, by depth migrating post-stack data after time demigration (Kim et al., 1997), or by direct mapping from time to depth. Each of these options requires converting the time migration velocity to a velocity model in depth. In the case of a laterally homogeneous medium, the conversion from time to depth is provided by the classic Dix method (Dix, 1955). However, the Dix conversion is not sufficient in the general case.

The theoretical connection between time migration velocities and true depth velocity models is provided by the concept of *image rays*, introduced by Hubral (1977). Image rays are seismic rays arriving normal to the surface of the earth. Hubral's theory explains how to model time migration velocities given the depth velocity model. However, it does not provide a convenient form for developing an accurate inversion algorithm. Moreover, tracing image rays is a numerically inconvenient procedure for achieving uniform coverage of the subsurface. This may explain why simplified image-ray tracing algorithms (Larner et al., 1981; Hatton et al., 1981) did not find widespread application in practice.

In this paper, we develop a new method for time-to-depth conversion of time-migrated images. Our method is based on the image ray theory but establishes a new ray-theoretic connection between time-migration velocities and interval seismic velocities. One can regard this connection as a natural extension of the classical Dix formula (Dix, 1955) to laterally inhomogeneous media. We invert the forward modeling of time migration velocities to produce two outputs: a time-migrated image mapped directly onto a uniform depth-domain grid and the interval seismic velocity model defined on the same grid. One can utilize the interval velocity further for depth imaging and for refined model building in the depth domain. We illustrate an application of our method on synthetic and field data examples.

FUNDAMENTALS OF TIME MIGRATION

Seismic reflection imaging can be understood in geometrical (ray-theoretical) terms with the help of the so-called *Kirchhoff prestack depth migration* operator. If $I(\mathbf{x})$ is the seismic image of the subsurface $\mathbf{x} = \{x,y,z\}$, and $D(t,\mathbf{s},\mathbf{r})$ is the reflection seismic data collected at the source position $\mathbf{s}$, receiver position $\mathbf{r}$ and time t, then the Kirchhoff imaging operator is

$$I(\mathbf{x}) = \iint \frac{\partial}{\partial t} D\left[T(\mathbf{x},\mathbf{s}) + T(\mathbf{x},\mathbf{r}),\mathbf{s},\mathbf{r}\right] A(\mathbf{x},\mathbf{s},\mathbf{r})\,\mathbf{ds}\,\mathbf{dr}\,, \qquad (1)$$

where $A(\mathbf{x},\mathbf{s},\mathbf{r})$ is the amplitude weight, and $T(\mathbf{x},\mathbf{y})$ is the traveltime between the subsurface point $\mathbf{x}$ and point $\mathbf{y}$ at the surface of the observations. The Kirchhoff migration operator can be derived from asymptotic inversion of the Born scattering approximation (Miller et al., 1987; Bleistein et al., 2001), from inversion of the Kirchhoff-Helmholtz integral (Tygel et al., 2001), or from geometrical considerations (Tygel et al., 1996). It order to implement operator (1), it is necessary to define the background velocity model for computing the traveltime and amplitude functions. The connection between traveltime and velocity is given by the eikonal equation, which, in the case of isotropic wave propagation, takes the form

$$|\nabla_{\mathbf{x}} T| = 1/v(\mathbf{x})\,, \qquad (2)$$

where $\nabla_{\mathbf{x}}$ denotes the gradient with respect to $\mathbf{x}$, and $v(\mathbf{x})$ is the depth velocity model.

Time migration avoids the need for an interval velocity model by making approximations. It approximates the traveltime function in equation (1) as

$$T(\mathbf{x},\mathbf{s}) + T(\mathbf{x},\mathbf{r}) \approx \widehat{T}(t_0,\mathbf{x}_0,\mathbf{s},\mathbf{r}) \qquad (3)$$

where t_0 and $\mathbf{x}_0$ are effective parameters of the subsurface point $\mathbf{x}$, and $\widehat{T}$ is an approximation, which usually takes the hyperbolic form

$$\widehat{T}(t_0,\mathbf{x}_0,\mathbf{s},\mathbf{r}) = \sqrt{t_0^2 + \frac{|\mathbf{x}_0 - \mathbf{s}|^2}{v_m^2(t_0,\mathbf{x}_0)}} + \sqrt{t_0^2 + \frac{|\mathbf{x}_0 - \mathbf{r}|^2}{v_m^2(t_0,\mathbf{x}_0)}}\,, \qquad (4)$$

although more complex nonhyperbolic forms are possible. Thus, the *Kirchhoff prestack time migration* operator defines a seismic image in the parameter space $\{t_0,\mathbf{x}_0\}$, as follows:

$$\widehat{I}(t_0,\mathbf{x}_0) = \iint \frac{\partial}{\partial t} D\left[\widehat{T}(t_0,\mathbf{x}_0,\mathbf{s},\mathbf{r}),\mathbf{s},\mathbf{r}\right] \widehat{A}(t_0,\mathbf{x}_0,\mathbf{s},\mathbf{r})\,\mathbf{ds}\,\mathbf{dr}\,. \qquad (5)$$

The goal of this paper is to construct a mapping from time migration coordinates $\{t_0,\mathbf{x}_0\}$ to the true reflection coordinates $\mathbf{x}$ and from time-migration velocity $v_m(t_0,\mathbf{x}_0)$ to the true interval velocity $v(\mathbf{x})$.

Approximation (4) can be understood as the truncated Taylor series for the squared traveltime around the surface point $\mathbf{x}_0$, where the ray connecting points $\mathbf{x}$ and $\mathbf{x}_0$ arrives normal to the surface. This is the *image ray* introduced by Hubral (1977). In the case of a constant velocity, the hyperbolic approximation is exact, the image ray is vertical, and time migration velocity coincides with the true velocity. In this case, time to depth mapping reduces to multiplying image time t_0 by velocity v_m. In the case of seismic velocity varying with depth only, the time migration velocity corresponds to the root mean square velocity, and one can recover the true velocity by simple differentiation (Dix, 1955).

In the next section, we establish a theoretical connection between time migration velocities and true velocities in the case of lateral velocity variations and non-vertical image rays.

FORWARD MODELING OF TIME MIGRATION VELOCITIES

For simplicity, from now on we will deal with a 2-D earth model. The results are fully extensible to 3-D.

Consider a small tube of rays. Pick some ray among them and create an orthogonal coordinate system $\{t,q\}$ attached to it. We will call this ray *central*. One can show (Popov and Pšenčik, 1978; Červený, 2001) that the equations of motions in the Hamiltonian form for the rays in the tube are

$$\frac{dq}{dt} = v_0^2\, p\,, \qquad \frac{dp}{dt} = -\frac{v_{nn}}{v_0}\, q. \tag{6}$$

Here v_0 is the velocity on the central ray, and v_{nn} is the second derivative of the velocity in the directions perpendicular to the central ray. Let α be some parameter of the ray tube. Introduce the following notations:

$$Q \equiv \frac{\partial q}{\partial \alpha}, \qquad P \equiv \frac{\partial p}{\partial \alpha}.$$

The quantities Q and P satisfy the equations in variations for equations (6):

$$\frac{dQ}{dt} = v_0^2\, P\,, \qquad \frac{dP}{dt} = -\frac{v_{nn}}{v_0}\, Q. \tag{7}$$

The quantity Q has a nice geometrical meaning. Its absolute value is the derivative of the length of the orthogonal cross section of the small tube of rays with respect to α. This derivative is called the *geometrical spreading*.

There are two important cases:

- Let the ray tube start at the surface perpendicular to it. Then pick $\alpha = x_0$, where x_0 is the coordinate of the starting point. This is the ray tube of image rays. We will call this ray tube the *telescopic family*.

- Let the ray tube come out of a source point $\{x_s, z_s\}$. Then pick α as the initial angles of the ray. We will call this ray tube the *source point family*.

Now consider an image ray arriving to the surface at a point x_0 and image rays around it. Suppose that we trace the central ray backward for time t_0, compute the quantities Q and P along it for the telescopic family of rays and reach a subsurface point $\{x,z\}$. We establish the following connection between the migration velocity $v_m(t_0,x_0)$ and the velocity at the subsurface point $\{x,z\}$ reached by the image ray:

$$\boxed{\frac{v(x,z)}{|Q(x,z)|} = \sqrt{\frac{\partial\left(t_0\, v_m^2(t_0,x_0)\right)}{\partial t_0}} = v_{Dix}(t_0,x_0)} \tag{8}$$

Here $v(x,z)$ and $Q(x,z)$ are the velocity and geometrical spreading of the telescopic family, respectively, at the point reached by a ray starting at the surface point x_0 perpendicular to the surface and having traveled for time t_0. Derivation of equation (8) is sketched in the appendix. When the image rays remain vertical and do not spread, $Q=1$, and the classic Dix method applies. In a more general case, the situation is different.

In the case of a laterally inhomogeneous medium, the Dix velocity is the true velocity divided by the geometrical spreading of image rays.

This connection can be expressed in the form of partial differential equations. Consider the mapping between Cartesian coordinates x,z and image ray coordinates (x_0,t_0). Functions $x_0(x,z)$ and $t_0(x,z)$ satisfy the following system of equations:

$$|\nabla x_0|^2 = \left(\frac{\partial x_0}{\partial x}\right)^2 + \left(\frac{\partial x_0}{\partial z}\right)^2 = \frac{1}{Q^2(x,z)}, \tag{9}$$

$$\nabla x_0 \cdot \nabla t_0 = \frac{\partial x_0}{\partial x}\frac{\partial t_0}{\partial x} + \frac{\partial x_0}{\partial z}\frac{\partial t_0}{\partial z} = 0, \tag{10}$$

$$|\nabla t_0|^2 = \left(\frac{\partial t_0}{\partial x}\right)^2 + \left(\frac{\partial t_0}{\partial z}\right)^2 = \frac{1}{v^2(x,z)} \tag{11}$$

with boundary conditions $x_0(x,0) = x, \quad t_0(x,0) = 0$. The task of time to depth conversion of time migrated images requires estimating all quantities in system (9)-(11) given the values of

$$v_{Dix}^2(x_0,t_0) = \frac{v^2\left(x(x_0,t_0), z(x_0,t_0)\right)}{Q^2\left(x(x_0,t_0), z(x_0,t_0)\right)}$$

for all surface points x_0 and times t_0.

INVERSION METHODS

In view of equation (8), we can state the following inverse problem. Suppose there is an image ray arriving at each surface point x_0, $x_{min} \le x_0 \le x_{max}$. For any $0 \le t_0 \le t_{max}$ trace the image ray backward for time t_0 together with the telescopic family of rays. Let the image ray reach a point (x,z). Denote by $v(x_0,t_0)$ the velocity at the point $\{x,z\}$, and by $Q(x_0,t_0)$ the quantity Q for the corresponding telescopic family at the point (x,z). We are given $v_{Dix}(x_0,t_0) = v(x_0,t_0)/|Q(x_0,t_0)|$, $x_{min} \le x_0 \le x_{max}$, $0 \le t_0 \le t_{max}$. We need to find $v(x,z)$, the velocity inside the domain covered with the image rays arriving to the surface in the interval $[x_{min}, x_{max}]$.

We introduce two methods for solving the inverse problem above. One is based on image ray tracing, and the second on the level set methods and fast marching methods (Osher and Sethian, 1988; Sethian, 1996, 1999). We are also working on a fast marching type method for solving the problem in the Eulerian formulation (9)-(11).

The ray tracing approach consists of three steps. First we compute the image rays solving the ray tracing system together with equations (7) for Q and P. Second, we recompute $Q(x_0,t_0)$ and the velocity $v(x_0,t_0)$ using the image rays found, as this significantly reduces the error. Third, we compute the velocity $v(x,z)$ from the velocity $v(x_0,t_0)$ solving system (10)-(11) with a fast marching type algorithm.

The level set type algorithm is the following. We propagate the wave front coinciding with the flat surface at $t=0$ downward the earth. We embed the wave front into a 2-D function $\phi(x,z)$ so that the front is its zero level set. Furthermore, we embed the quantities Q and P defined on the front into 2-D functions $q(x,z)$ and $p(x,z)$ so that at each moment of time Q and P coincide with q and p on the zero level set of $\phi(x,z)$. The functions $\phi(x,z)$, $q(x,z)$ and $p(x,z)$ satisfy the following system of partial differential equations:

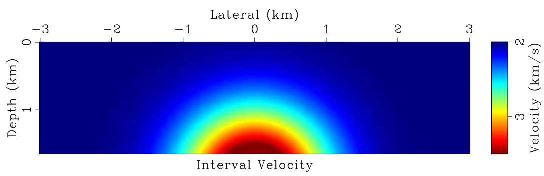
$$\phi_{t_0} + v(x,z)|\nabla\phi| = 0\,, \quad q_{t_0} = v^2(x,z)\, p\,,$$
$$p_{t_0} = \frac{v_{xx}\, g_x^2 - 2\, v_{xz}\, g_x\, g_z + v_{zz}\, g_z^2}{v(x,z)}\, q\,, \tag{12}$$

where $g_x = \frac{\phi_x}{\nabla\phi}, \quad g_z = \frac{\phi_z}{\nabla\phi}$.

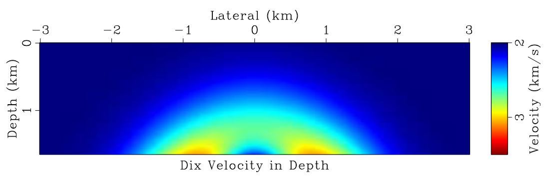
We initialize $q(x,z)=1$, $p(x,z)=0$, $\phi(x,z)=z$ which is correct for $t=0$ and solve system (12) by iterating the following steps. First we find the velocity $v(x,z)$ for the current $q(x,z)$. Then we extract the front, find the velocity on the front and build the extension velocity. Finally we update ϕ, q and p making a time step in system (12) using the extension velocity. Note that the extension velocity is built in such a way that ϕ remains the signed distance from the front (Sethian, 1999).

SYNTHETIC DATA EXAMPLE

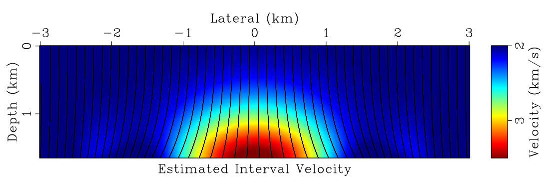
Figure 1(a) shows a synthetic velocity model with a Gaussian velocity anomaly. The corresponding Dix velocity mapped from time to depth is shown in Figure 1(b). There is a significant difference between both the value and the shape of the velocity anomaly recovered by the Dix method and the true anomaly. The difference is explained by taking into account geometrical spreading of image rays. Figure 1(c) shows the velocity recovered by our method and the corresponding family of image rays.



(a)



(b)



(c)

Figure 1: Synthetic test on interval velocity estimation. (a) Exact velocity model. (b) Dix velocity converted to depth. (c) Estimated velocity model and the corresponding image rays. The image ray spreading causes significant differences between Dix velocities and true velocities.

FIELD DATA EXAMPLE

Figure 2, taken from (Fomel, 2003), shows a prestack time migrated image from the North Sea and the corresponding time migration velocity obtained by velocity continuation. The most prominent feature in the image is a salt body which causes significant lateral variations of velocity. Figure 3 shows the top portion of the interval velocity model recovered by our method and the corresponding image rays. We stopped image ray tracing at the point when the algorithm error started to increase. A good strategy for a complicated velocity model like this one is imaging with redatuming (Bevc, 1997) and iterations between time and depth migration. Figure 4 compares two images: a prestack time migration image converted to depth with our algorithm and a post-stack depth migration image using the estimated velocity model. A good structural agreement between the two images is an indirect evidence of the algorithm success. An ultimate validation can come from prestack depth migration velocity analysis, which is significantly more expensive.

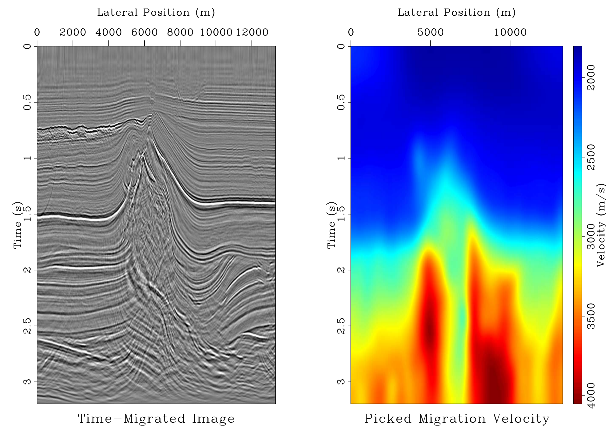


Figure 2: Left: seismic image from North Sea obtained by prestack time migration using velocity continuation (Fomel, 2003). Right: corresponding time migration velocity.

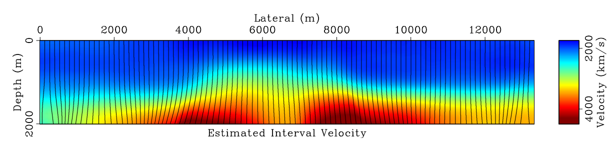


Figure 3: Estimated depth velocity model and the corresponding image rays.

CONCLUSIONS

We have proved that the Dix velocity obtainable from the time migration velocity is the true velocity divided by the geometrical spreading of image rays. We have posed the corresponding inverse problem and suggested two algorithms for solving it. We tested these algorithms on a synthetic data example with laterally heterogeneous velocity and demonstrated that they produce significantly better results than simple Dix inversion followed by time-to-depth conversion. Moreover, the Dix velocity may qualitatively differ from the output velocity. We also tested our algorithm on a real data example and validated our algorithms by comparing prestack time migration image mapped to depth with a post-stack depth migrated image.

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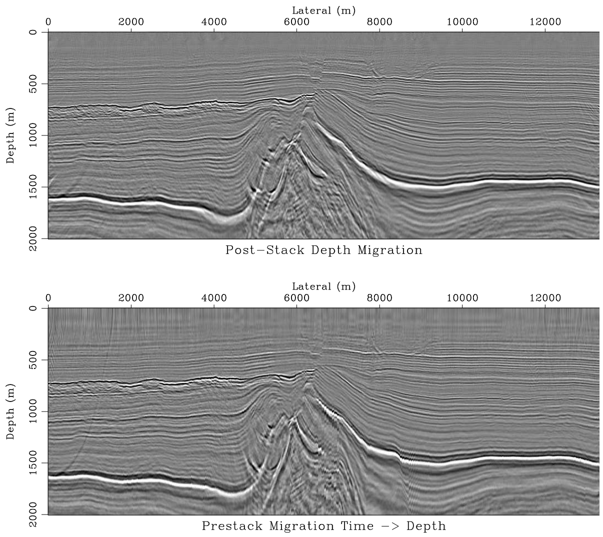

Figure 4: Top: prestack time migrated image converted to depth with our method. Bottom: post-stack depth migration using the interval velocity model estimated by our method.

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APPENDIX A

CONNECTION BETWEEN DIX VELOCITIES AND INTERVAL VELOCITIES IN A LATERALLY HETEROGENEOUS MEDIUM

In this appendix, we sketch the proof of formula (8). The detailed proof can be found in (Cameron, 2007) and (Cameron et al., 2006).

Let us consider the quantity $K = vR$ along the image ray, where v is the velocity along the image ray, and R is the radius of curvature of the wave front for the source point family of rays. Suppose the image ray passes the point $\{x,z\}$ at time t_0 and arrives to the surface at time t_1. Using equation (4), one can show that

$$K(t_1-t_0,x_0) = (t_1-t_0)\,v_m^2(t_1-t_0,x_0), \qquad \text{(A-1)}$$

where v_m is the migration velocity given as a function of the surface point x_0 and the one-way travel time.

Thus, we can find the values of $K(t_1-t_0,x_0)$ for all x_0 and t_1-t_0 from the migration velocities. Popov and Pšenčik (1978) showed that, for a source family of rays,

$$K_t = v^2 + \frac{v_{nn}}{v}K^2, \qquad K(t_0) = 0 \qquad \text{(A-2)}$$

and that K can be decomposed into the ratio of Q and P: $K = Q/P$.

Introduce the following notations: $X = (Q,P)^T$, $A(t) = (v^2(t), v_{nn}/v)^T$. Let X_* be a matrix of derivatives of X with respect to the initial values of Q and P, Q_0 and P_0 respectively. For the source point family of rays starting at time t_0 write the initial value problems for Q and P and their derivatives with respect to the initial data in terms of X, X_* and $A(t)$:

$$\frac{dX}{dt} = A(t)X, \qquad Q(t_0) = 0, \quad P(t_0) = \frac{1}{v(t_0)}, \qquad \text{(A-3)}$$

$$\frac{dX_*}{dt} = A(t)X_*, \qquad X_*(t_0) = I\,, \qquad \text{(A-4)}$$

where I is the identity matrix. The solution of (A-4) at time t_1 is the propagator matrix $B(t_0;t_1)$ in notation of Červený (2001), and the solution of (A-3) is: $Q(t_1) = B_{12}(t_1)/v(t_0)$, $P(t_1) = B_{22}(t_1)/v(t_0)$, where B_{12} and B_{22} are the entries of the matrix $B(t_0;t_1)$. Now turn to the quantity $K = vR = Q/P$. For our source point family of rays, $K(t_0;t_1) = Q/P = B_{12}/B_{22}$. Let us express the derivatives of K with respect to the initial values of Q and P in terms of the entries of the matrix X_*:

$$\frac{\partial K}{\partial Q_0} = \frac{\partial Q}{\partial Q_0}\frac{1}{P} - \frac{\partial P}{\partial Q_0}\frac{Q}{P^2}, \qquad \frac{\partial K}{\partial P_0} = \frac{\partial Q}{\partial P_0}\frac{1}{P} - \frac{\partial P}{\partial P_0}\frac{Q}{P^2}\,. \qquad \text{(A-5)}$$

The initial data for Q and P at time t_0 are $Q_0 = Q(t_0;t_0) = 0$, $P_0 = P(t_0;t_0) = 1/v(t_0)$. Now shift the initial moment of time and make it $t_0 - \Delta t_0$. If the initial data for Q and P at $t_0 - \Delta t_0$ are 0 and $1/v(t_0 - \Delta t_0))$ respectively, then at time t_0 we have:

$$\begin{pmatrix} Q_0 + \Delta Q_0 \\ P_0 + \Delta P_0 \end{pmatrix} = \begin{pmatrix} v(t_0)\Delta t_0 \\ \frac{1}{v(t_0)} \end{pmatrix} + O((\Delta t_0)^2)\,. \qquad \text{(A-6)}$$

Extracting ΔQ_0 and ΔP_0 from here and using relations (A-5) we find the value of K for the wave front started at time $t_0 - \Delta t_0$ at time t_1, i.e, at the surface:

$$K(t_0-\Delta t_0;t_1) = K(t_0;t_1) + \frac{\partial K}{\partial Q_0}\Delta Q_0 + \frac{\partial K}{\partial P_0}\Delta P_0\,. \qquad \text{(A-7)}$$

Rewriting equation (A-7) in terms of the entries of the propagator matrix $B(t_0;t_1)$ and taken into account that $B_{12} = Q\,v(t_0)$, $B_{22} = P\,v(t_0)$, and $\det B = 1 = (B_{11}\,P - B_{21}\,Q)\,v(t_0)$ we find the derivative of K at the surface with respect to the initial moment of time t_0:

$$-\frac{\partial K(t_0;t_1)}{\partial t_0} = \frac{1}{P^2}\,. \qquad \text{(A-8)}$$

Using the reciprocity of Q and P (Červený, 2001) and equation (A-1) and returning to the notation t_0 for the one-way travel time along the image ray we get formula (8).

EDITED REFERENCES

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