PRACTICE MIDTERMS

Test 1.
1. Evaluate the following integrals:

(a) \[ \int \sin 2x \sin x \, dx \]

(b) \[ \int \frac{dt}{t^2 + t^3} \]

(c) \[ \int \frac{x^{1/2} \, dx}{1 + x^{1/4}} \]

(d) \[ \int \cos \sqrt{x} \, dx \]

2. Determine whether each improper integral is convergent or divergent. Evaluate the integrals which are convergent:

(a) \[ \int_{-1}^{1} \frac{dx}{x^3} \]

(b) \[ \int_{0}^{\infty} xe^{-x} \, dx \]

3. Determine how large do we have to choose \( n \) to evaluate

\[ \int_{-2}^{-1} e^{1/x} \, dx \]

with an error less than 0.01 using the trapezoidal rule. Write formula for this approximation. Do not evaluate! Use the formula \( |E_T| \leq \frac{(b-a)^3K}{12n^2} \) if \( |f''(x)| \leq K \) for \( a \leq x \leq b. \)

4. Determine whether the integral

\[ \int_{-\infty}^{\infty} e^{-x^2 - 2x} \, dx \]

is convergent or divergent. Justify your answer. Do not try to evaluate this integral!

Test 2.
1. Evaluate the following integrals:

(a) \[ \int \sin^3 x \cos x \, dx \]

(b) \[ \int \frac{x^3}{(x-1)^3} \, dx \]
(c) $\int \frac{dx}{e^{2x} + 3e^x + 2}$
(d) $\int (4 - x^2)^{1/2} \, dx$

2. Let

$$I_n = \int_0^{\pi/2} \sin^n x \, dx.$$  

Show that for $n > 1$

$$I_n = \frac{n-1}{n} I_{n-2}.$$ 

3. Determine how large do we have to choose $n$ to evaluate

$$\int_0^{10} \cos x^2 \, dx$$

with an error less than $10^{-5}$ using the midpoint rule. Use the formula $|E_M| \leq \frac{(b-a)^3 K}{24n^2}$ if $|f''(x)| \leq K$ for $a \leq x \leq b$.

4. Determine whether the integral

$$\int_0^{\pi/2} \sec x \, dx$$

is convergent. Justify your answer.

Test 3. 1. Evaluate the following integrals:

(a) $\int \frac{\ln x}{x (\ln x + 1)} \, dx$
(b) $\int \frac{x^2}{(x + 1)^{2008}} \, dx$
(c) $\int \frac{1 - \tan x}{1 + \tan x} \, dx$

2. Evaluate the area under the curve $y = e^{2x} \sin x$, $0 \leq x \leq \pi$.

3. Estimate the error of evaluating the integral

$$\int_0^{x=1} \cos x^2 \, dx$$

using the midpoint rule with $n = 100$.

4. Determine whether the integral

$$\int_0^{\infty} \frac{dx}{x^2 + 4x + 3}$$

is convergent. Justify your answer. If the integral is convergent, evaluate it.