SAMPLE MIDTERMS
MATH 55

Sample 1.

1. Here is the definition of a strictly increasing function $f : \mathbb{R} \to \mathbb{R}$:

$$\forall x, y \in \mathbb{R} \quad (x < y \to f(x) < f(y)).$$

Write in the similar way the statement that f is not strictly increasing.

2. Which of the following sets are infinitely countable

(a) $\mathbb{Z} \times \mathbb{Z}$,

(b) $\mathbb{R} - \mathbb{Z}$,

(c) the set of all irrational real numbers?

Justify your answer.

3. Find the binary and hexadecimal expansions for 500.

4. Solve the following system of congruence equations

$$x \equiv 3 (\mod 5)$$
$$x \equiv 4 (\mod 7)$$
$$x \equiv 2 (\mod 9).$$

5. Prove by induction the identity

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}.$$

Sample 2.

1. Prove that for three sets A, B and C, $|A| = |B|$ and $|B| = |C|$ implies $|A| = |C|$.

2. A sequence a_n is *strictly* increasing if for all n, $a_{n+1} > a_n$. Let a_n be given be given recursively

$$a_1 = 1, \; a_n = a_{n-1}^2 + a_{n-1} \text{ for } n = 2, 3, 4, \dots$$

(a) Write first five terms of the sequence.

(b) Prove that a_n is strictly increasing.

3. Find $gcd(2012, 344)$ using Euclidean algorithm.

4. Find all prime p such that $p^2 + 14$ is also prime. (Hint: use arithmetic modulo 3.)

5. Let $A_n = \{nk \,|\, k \in \mathbb{Z}\}$. Find

$$\bigcap_{n=1}^{\infty} A_n.$$

Sample 3

1. Let $f : A \to B$ be a surjective function. Prove that $|B| \leq |A|$.

2. Prove that the number is divisible by 9 iff the sum of its digits is divisible by 9.

3. Using strong induction prove that any positive integer can be written as a sum of distinct Fibonacci numbers.

4. Let $P(n)$ denote the predicate $n^2 + n + 41$ is prime.

(a) Find the truth values of $P(1)$, $P(2)$, $P(5)$.

(b) Find n such that $P(n)$ is false.

5. Write in pseudocode the procedure which finds a primitive root modulo a prime p.