Due November 4.

1. Let \( R \) be the algebra of polynomial differential operators. In other words \( R \) is generated by \( x \) and \( \frac{\partial}{\partial x} \) with relation

\[
\frac{\partial}{\partial x} x - x \frac{\partial}{\partial x} = 1.
\]

(The algebra \( R \) is called the Weyl algebra.) Let \( M = \mathbb{C}_x \) have a structure of \( R \)-module in the natural way. Show that \( \text{End}_R(M) = \mathbb{C} \), \( M \) is an irreducible \( R \)-module and the natural map \( R \to \text{End}_\mathbb{C}(M) \) is not surjective.

2. Let \( R \) be a subalgebra of upper triangular matrices in \( \text{Mat}_n(\mathbb{C}) \). Classify simple and indecomposable projective modules over \( R \) and evaluate \( \text{Ext}_R(M,N) \) for all simple \( M \) and \( N \).

\textit{Date:} October 27, 2005.