PROBLEM SET # 5
MATH 252

Due October 7.

1. Classify irreducible representations of $A_4$ (even permutations) over $\mathbb{R}$ and over $\mathbb{C}$. What is the splitting field for $A_4$.

2. Let $G$ be a finite group, $r$ be the number of conjugacy classes in $G$ and $s$ be the number of conjugacy classes in $G$ preserved by the involution $g \rightarrow g^{-1}$. Prove that the number of irreducible representations of $G$ over $\mathbb{R}$ is equal to $\frac{r+s}{2}$.

3. If $\lambda$ is a Young tableau, then the conjugate tableau $\lambda'$ is obtained from $\lambda$ by symmetry about diagonal (rows and columns switch). Show that $V_{\lambda'}$ is isomorphic to $V_{\lambda} \otimes \text{sgn}$, where sgn is one-dimensional sign representation. (Hint: you probably have to show that $\mathbb{Q}(S_n) a_{\lambda} b_{\lambda}$ and $\mathbb{Q}(S_n) b_{\lambda} a_{\lambda}$ are isomorphic).

Date: September 29, 2005.