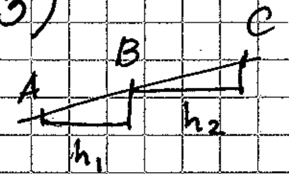
So we choose $b_1 = a_1 \pm h$

$b_2 = a_1 \pm mh$

(C2) The equivalence relation is straightforward

$d(AB) = d(CD) = d$, $d(EF) = d(CD) = d \Rightarrow d(AB) = d(EF) = d$.

(C3)

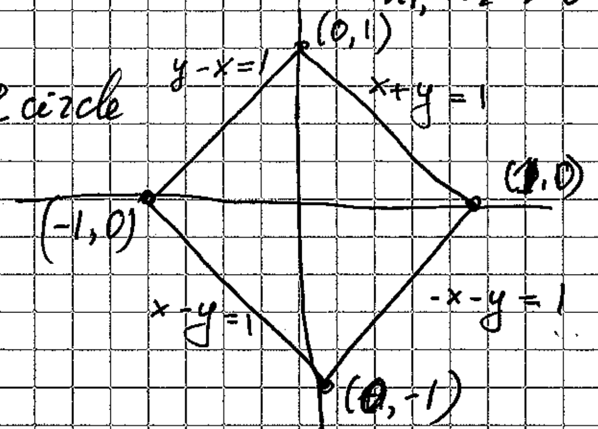


$d(AC) = |h_1|(1+|m|) + |h_2|(1+|m|) =$

$= (h_1 + h_2)(1+|m|) = d(AB) + d(BC)$.

$h_1, h_2 > 0$ or $h_1, h_2 < 0$

The circle



8.8 (C1) If for the slope of a line (AB) is m and $|m| \le 1$ Then $d(A,B) = |b_1 - a_1|$

if $|m| \ge 1$, the $d(A,B) = |b_2 - a_2|$

So in the first case take $b_1 = a_1 \pm d$

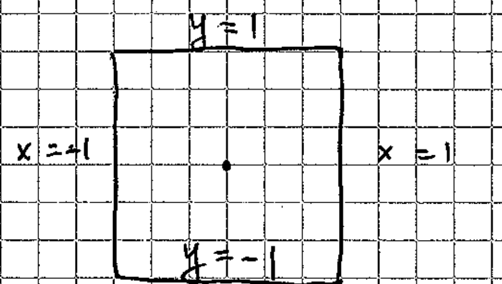
in the second case take $b_2 = a_2 \pm \frac{d}{m}$

(C2) as in the previous problem.

(C3) Again we have to check $d(AC) = d(AB) + d(BC)$ if $A * B * C$. So check two cases the slope 1) $|m| \le 1$ $d(AC) = |c_1 - a_1| = |c_1 - b_1| + |b_1 - a_1|$

2) $|m| \ge 1$ $d(AC) = |c_2 - a_2| = |c_2 - b_2| + |b_2 - a_2|$

The circle



8.3 $AB + EF \cong EF + AB$, $CD + EF \cong EF + CD$

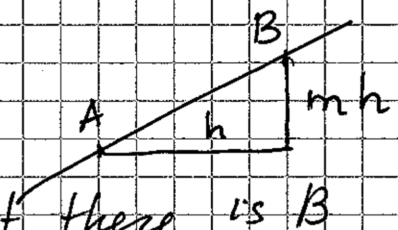
By (C1)

on the ray opposite to $\overrightarrow{FE}$ construct G and H so that $FG \cong AB$ and $FH \cong CD$. Then $FG < FH$. So $F * G * H$. But then $E * G * H$ So $EG < EH$. Since $EG \cong EF + AB$, $EH \cong EF + CD$, we have $AB + EF \cong CD + EF$.

8.5 (a) Let ℓ be a line containing O. On each ray of ℓ originating at O there is a unique point X such that $OX \cong OA$ (by (C1)). There are two rays, hence there are two points.

(b) Let m be a line that does not contain O. By the statement proven in class m has infinitely many points. Therefore for each $X \in m$ there exists an unique $Y \in \overrightarrow{OX}$ such that $OY \cong OA$. Thus, we have infinitely many points Y on Γ.

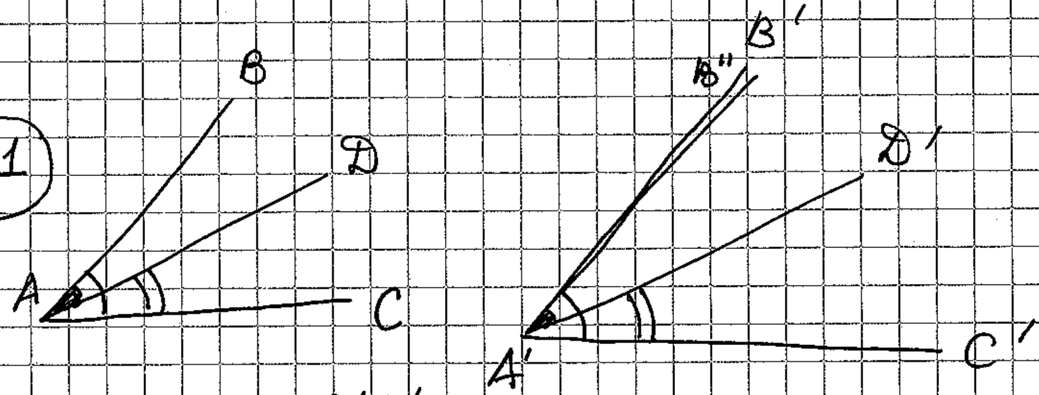
8.7 (C1) Let d be a positive real number.



If there is a line through A with slope m. We have to show that there is B on each side such that $d(AB) = d$

$|h|(1+|m|) = d$ $\quad h = \pm \frac{d}{1+|m|}$

9.1



Given $\angle BAC \cong \angle B'A'C'$

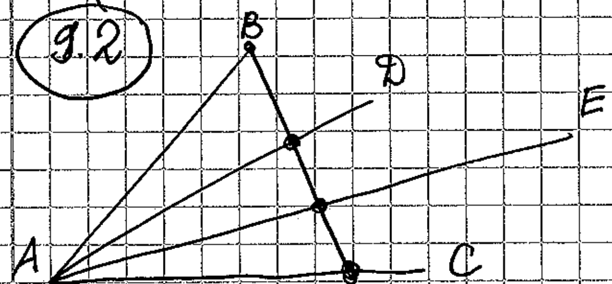
By (C4) there exists a ray $\overrightarrow{A'D'}$ on the same side of $A'C'$ as B' such that $\angle D'A'C' \cong \angle DAC$.

Then on the same side of $A'D'$ as B' there exist $\overrightarrow{A'B''}$ such that $\angle B''A'D' \cong \angle BAD$.

Now we use Proposition 9.4 (addition of angles)

$\angle BAC \cong \angle B''A'C' \cong \angle B'A'C'$. By uniqueness part of (C4) $\angle B'A'C' = \angle B''A'C'$ so $\overrightarrow{A'B'} = \overrightarrow{A'B''}$.

9.2



B and D are on the same side of AC, and D and E are on the same side of AC. So B and E are on the same side of AC. C and D are on the same side of AB, and C and E are on the same side of AB. By Crossbar theorem, we may assume that $B * D * C$, $D * E * C$. So we have $\overrightarrow{DE} = \overrightarrow{DC}$ and $\overrightarrow{DB}$ is the opposite ray. So $B * E * C$. So E and C are on the same side of AB.

8.9 Let $\varphi: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map $\varphi(a_1, a_2) = (a_1 + a_2, a_1 - a_2)$. Then φ preserves ~~[illegible]~~ betweenness and maps a line to a line, because φ is a linear map. If d_1, d_2 are distance functions in examples 8.7 and 8.8 respectively then

$$d_2(\varphi(A), \varphi(B)) = \sup\{|a_1 + a_2 - b_1 - b_2|, \{|a_1 - a_2 - b_1 + b_2|\}$$
$$= \sup\{|(a_1 - b_1) + (a_2 - b_2)|, |(a_1 - b_1) - (a_2 - b_2)|\} =$$
$$= |a_1 - b_1| + |a_2 - b_2| = d_1(A, B)$$

Hence congruent segments go to congruent segment, and φ is an isomorphism.

To show that the Standard model is not isomorphic note that in the standard model a circle and a line meet at most in two points.

But in examples 8.7, 8.8 a circle can intersect a line at the segment.