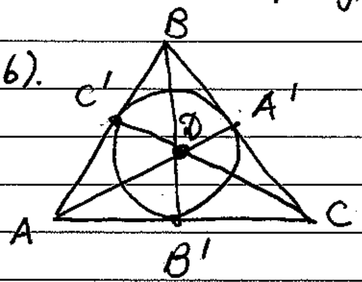
(6.3)

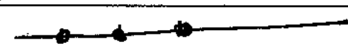
a) There are 3 noncollinear points A, B, C
Then line AB containts at least one more point C', similarly BC has a point A' and AC has a point B', C' does not belong to BC or AC, similarly B' does not belong to AB or BC, A' does not belong to AC or AB. Therefore A', B', C', A, B, C are distinct points.

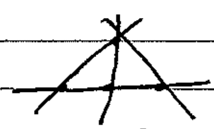
The line CC' does not contain A, B, A' or B'
By $P3$ it must contain at least one more point D.
Thus, a projective plane has at least 7 points.

b). 

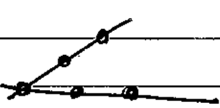
This is a model with 7 points. We just observe that CC', BB' and AA' must meet at one point, and the line $A'B'$ must have one more point which can only be C.

c). ~~$P(1) P(2) P(3) \not\Rightarrow P(4)$~~

$P1, P2, P3 \not\Rightarrow P4$ 

$P1, P2, P4 \not\Rightarrow P3$ 

$P1, P3, P4 \not\Rightarrow P2$ $\mathbb{R}^2$

$P2, P3, P4 \not\Rightarrow P1$ 

(6.7) (a) (P1) if both ~~lines~~ points are not ideal (P1) follows from (I1) for Π

if A is ideal, B is not then the line containing A and B is a line through B ~~containing~~ which is contained in a pencil corresponding to A.

if both A and B are ideal points line AB must be the ~~ideal~~ line at infinity

(P2) If l_1 and l_2 are both not a line at infinity then either ~~l_1 and l_2~~ $l_1 \cap l_2 \cap \Pi = \emptyset$ or $l_1 \cap l_2 \cap \Pi \neq \emptyset$. In the first case l_1 and l_2 lie in the same pencil, hence they meet at the ideal point corresponding to this pencil. In the second case $l_1 \cap l_2 \neq \emptyset$ already.

(P3) Let l be a line. If l is not the line at infinity, then $l \cap \Pi$ has at least 2 points and l also has one ideal point corresponding to the pencil of l. If l is a line at infinity pick 3 noncollinear points on Π, A, B and C. Then AB, BC and AC belong to 3 distinct pencils. Hence the line at infinity also has at least 3 point.

(P4) Follows from I3 for Π.

(b) Consider a plane $\Pi \subset \mathbb{F}^3$ which does not pass through ~~the~~ zero. Then for any point $x \in \Pi$ there is exactly one 1-dimensional ~~plane~~ space $\mathbb{F}x$ ~~in the~~ that is a "point" in Π'.

Let L be the two dimensional subspace in $\mathbb{F}^3$ parallel to Π. Then L is "the line at infinity". Parallel lines in Π have the same direction which is a 1-dimensional subspace in L, that is an "ideal point".

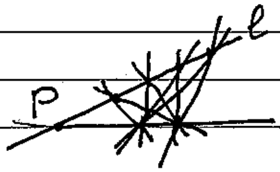
6.10. For each poin P let $m(P)$ be the number of lines containing P. For each line ℓ let $m(\ell)$ be the number of points on ℓ.

Let $a = \min\limits_{P} m(P)$, $b = \max\limits_{\ell} m(L)$. If n is the number of points and N is the number of lines, then $N \geq \frac{na}{b}$.

If $a \geq b$ then $N \geq n$.

Let $a < b$, P be a point such that $m(P) = a$, and ℓ be a line such that $m(\ell) = b$.

If $a < b$, then $P \in \ell$ (otherwese for each point X on ℓ there is a line PX).



Let m be another line containing P which has maximal possible number of points among all such lines. Let this number be c.

Then $N \geq (c-1)(b-1) + a$

$n \leq (c-1)(a-1) + b$

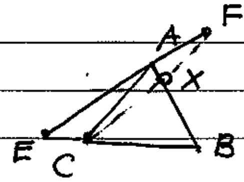
$N - n \geq (c-1)(b-a) + a - b = (c-2)(b-a) \geq 0$

since $b-a \geq 0$, $c \geq 2$.

(7.2) By line separation $C, D \in \overrightarrow{AB}$. Assume that $C * A * D$. Then $C \notin \overrightarrow{AD} = \overrightarrow{AB}$. Contradiction.

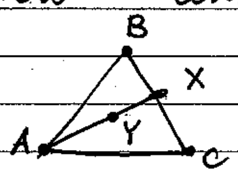
(7.9.) For any $A \neq B$ there is X such that $A * X * B$.

<u>Proof</u>. Take C not on AB, E so that $E * C * B$, F so that $E * A * F$. Then line CF meets $\overline{AE}$ or $\overline{AB}$ (B4).



But if CF meets $\overline{AE}$ it meets it at F. Thus, CF meet AB at X, $A * X * B$.

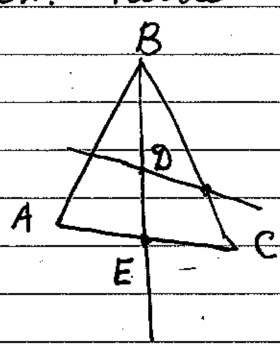
Now consider $\triangle ABC$.



Let $A * X * B$, and $A * Y * X$.
Then Y is on the same side of AB as X and as C. Similarly, Y is on the side of AC as B. Finally, Y is on the same side of BC as A. Thus, Y is inside $\triangle ABC$.

(7.10) An interior of a triangle is the intersection of 3 half planes. By ~~construction~~ definition each half plane is a convex set
is a convex set. An intersection of convex set is convex. Hence the statement.

(7.10)



Consider the ray $\overrightarrow{BD}$. By Crossbar theorem it meets $\overline{AC}$ at some point E.
If $\ell = BD$ the statement is proven. Let $\ell \neq BD$.
By (B4) any line ℓ containing D meets $\overline{BC}$ or $\overline{EC}$. Hence it meets a side of $\triangle ABC$.