

6.3

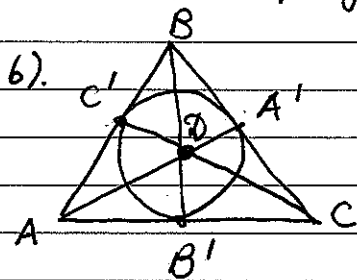
a) There are 3 noncollinear points A, B, C

Then line AB contains at least one more point C' , similarly BC has a point A' and AC has a point B' , C' does not belong to BC or AC , similarly B' does not belong to AB or BC , A' does not belong to AC or AB . Therefore A', B', C', A, B, C are distinct points.

The line CC' does not contain A, B, A' or B'

By P3 it must contain ^{at least} one more point D

Thus, a projective plane has at least 7 points.



b).

This is a model with 7 points.

We just observe that

CC', BB' and AA' must meet at one point, and the line $A'B'$ must have one more point which can only be C .

c). ~~$P_1, P_2, P_3 \Rightarrow P_4$~~

$P_1, P_2, P_3 \not\Rightarrow P_4$



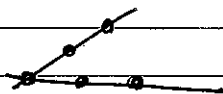
$P_1, P_2, P_4 \not\Rightarrow P_3$



$P_1, P_3, P_4 \not\Rightarrow P_2$

\mathbb{R}^2

$P_2, P_3, P_4 \not\Rightarrow P_1$



(6.4) (a) (P1) if both ~~lines~~ points are not ideal
(P1) follows from (I1) for Π

if A is ideal, B is not then the line containing A and B is a line through B ~~containing~~ which is contained in a pencil corresponding to A .

if both A and B are ideal points line AB must be the ~~ideal~~ line at infinity

(P2) If l_1 and l_2 are both not a line at infinity then either ~~they are~~ $l_1 \cap l_2 \cap \Pi = \emptyset$ or $l_1 \cap l_2 \cap \Pi \neq \emptyset$. In the first case l_1 and l_2 lie in the same pencil, hence they meet at the ideal point corresponding to this pencil. In the second case $l_1 \cap l_2 \neq \emptyset$ already.

(P3) Let l be a line. If l is not the line at infinity, then $l \cap \Pi$ has at least 2 points and l also has one ideal point corresponding to the pencil of l . If l is a line at infinity pick 3 noncollinear points on Π , A, B and C .

(P4) Then AB, BC and AC belong to 3 distinct pencils. Hence the line at infinity also has at least 3 point.

(P4) Follows from I3 for Π .

(b) Consider a plane $\Pi \subset \mathbb{F}^3$ which does not pass through ~~the~~ zero. Then for any point $x \in \Pi$ there is exactly one 1-dimensional ~~subspace~~ ^{space} $\mathbb{F}x$ ~~in the~~ that is a "point" on Π '.

~~Let~~ Let L be the two dimensional subspace in \mathbb{F}^3 parallel to Π . Then L is "the line at infinity". Parallel lines in Π have the same direction which is a 1-dimensional subspace in L , "that is an "ideal point".

6.10.

For each point P let $m(P)$ be the number of lines containing P . For each line l let $m(l)$ be the number of points on l .

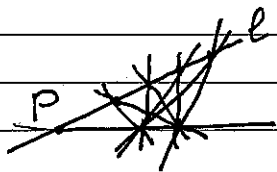
Let $a = \min_P m(P)$, $b = \max_l m(l)$. If n

is the number of points and N is the number of lines, then $N \geq \frac{na}{b}$.

If $a \geq b$ then $N \geq n$.

Let $a < b$, P be a point such that $m(P) = a$, and l be a line such that $m(l) = b$.

If $a < b$, then $P \in l$ (otherwise for each point X on l there is a line PX).



Let m be another line containing P which has maximal possible number of points among all such lines. Let this number be c .

Then $N \geq (c-1)(b-1) + \cancel{(c-1)(b-1)} + a$

$$n \leq (c-1)(a-1) + b$$

$$N - n \geq (c-1)(b-a) + a - b = (c-2)(b-a) \geq 0$$

since $b-a > 0$, $c \geq 2$.