PROBLEM SET # 5

Due February 27..

1. Show that the classical Lie algebras $sl(n,\mathbb{C})$ for $n \geq 2$, $so(n,\mathbb{C})$ for $n \geq 3$ and $sp(2n,\mathbb{C})$ for $n \geq 1$ are semisimple.

2. Let $\mathfrak{g}$ be a simple Lie algebra over $\mathbb{R}$ or $\mathbb{C}$. Prove that any two invariant symmetric forms on $\mathfrak{g}$ are proportional.

3. Let β be a non-degenerate symmetric invariant form on a finite-dimensional Lie algebra $\mathfrak{g}$. Let $\{u_1,\dots,u_n\}$ be a basis in $\mathfrak{g}$ and $\{u^1,\dots,u^n\}$ be the dual basis so that $\beta(u_i,u^j)=\delta_{i,j}$. Show that

$$\Omega = \sum_{i=1}^{n} u_i u^i$$

lies in the center of the universal enveloping algebra $U(\mathfrak{g})$.

4. Let $\mathfrak{g} = \mathfrak{sl}(2,\mathbb{C})$. Prove that the center of $U(\mathfrak{g})$ is generated by Ω. (Hint: use the standard basis e,h,f and prove first that the centralizer of h is generated by h and Ω.)

Date: February 20, 2017.