PROBLEM SET # 4

Due February 22.

1. Compute the terms of degree 3 and 4 in the Baker–Campbell–Hausdorff formula.

2. Use PBW theorem to prove that

(a) The universal enveloping algebra $U(\mathfrak{g})$ does not have zero divisors.

(b) If $\mathfrak{g}$ is a finite-dimensional Lie algebra, then $U(\mathfrak{g})$ is left and right Noetherian.

3. Let $\mathfrak{g}$ be a finite-dimensional nilpotent Lie algebra. Use Baker–Campbell–Hausdorff formula to prove existence of a simply connected Lie group G with Lie algebra $\mathfrak{g}$.

4. Prove that $U(\mathfrak{g})$ is a module over $\mathfrak{g}$ with action given by

$$\mathrm{ad}_g(u) = gu - ug$$

for all $g \in \mathfrak{g}$ and $u \in U(\mathfrak{g})$. Then check that for ground field of characteristic zero, the $\mathfrak{g}$-modules $U(\mathfrak{g})$ and $S(\mathfrak{g})$ are isomorphic by proving that the symmetrization map

$$S(\mathfrak{g}) \to T(\mathfrak{g}) \to U(\mathfrak{g})$$

is an isomorphism of $\mathfrak{g}$-modules.

Date: February 8, 2017.