PROBLEM SET # 1

Due January 130.

1. Show that $SO(n)$ is connected for all $n \geq 1$.

2. Describe the Lie algebras of $U(n)$ and $SU(n)$ as subalgebras in $M(n, \mathbb{C})$. Calculate dimensions of $U(n)$ and $SU(n)$.

3. Let Q be a non-degenerate symmetric form on $\mathbb{R}^n$ of signature (p, q), where $p+q = n$ i.e. there exists a Q-orthogonal basis $v_1, \ldots, v_n$ in $\mathbb{R}^n$ such that $Q(v_i, v_i) = 1$ for $i \leq p$ and $Q(v_i, v_i) = -1$ for $i > p$. Let $SO(p, q)$ be the subgroup of all operators $X \in SL(n, \mathbb{R})$ such that $Q(Xv, Xw) = Q(v, w)$ for all $v, w \in \mathbb{R}^n$.

(a) Calculate dimension of $SO(p, q)$.

(b) Prove that $SO(p, q)$ is compact if and only if $p = 0$ or $q = 0$.

(c) Show that $SO(1, n-1)$ has two connected components.

Date: January 16, 2017.