PROBLEM SET # 6

Due March 11.

1. Let $\mathfrak{h}$ denote the Heisenberg Lie algebra over $\mathbb{C}$ with basis $\{x, y, z\}$ and the bracket

$$[x, y] = z,\ [z, x] = [z, y] = 0.$$

Show that $\mathrm{Der}(\mathfrak{h})$ is isomorphic to a semidirect product of $\mathfrak{gl}(2)$ and the two dimensional abelian ideal, which is the image $\mathrm{ad}\,\mathfrak{h}$ in $\mathrm{Der}(\mathfrak{h})$.

2. Let k be a field of positive characteristic $p > 2$ and $S = k[x]/(x^p)$. Denote by W the Lie algebra of derivations of S.

(a) Show that W has dimension p and the basis $\{\frac{\partial}{\partial x}, x\frac{\partial}{\partial x}, \dots, x^{p-1}\frac{\partial}{\partial x}\}$. Compute the bracket in this basis.

(b) Prove that W is a simple Lie algebra.

(c) Prove that if $p > 3$, then the Killing form on W is trivial. What happens when $p = 3$?

3. In the Baker–Campbell–Hausdorff formula $F(x, y)$ denote the linear term in y. Show that

$$F(x, y) = \frac{\mathrm{ad}_x}{1 - e^{\mathrm{ad}_x}}(y).$$

For hints see problem 8 chapter 3 in the notes.

Date: March 2, 2020.